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Modelling and comparison of driverless and electric taxi operational modes: case study in Barcelona

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TREBALL FINAL DE MÀSTER

“Caminante, son tus huellas
el camino, y nada más;
caminante, no hay camino,
se hace camino al andar.
Al andar se hace camino,
y al volver la vista atrás
se ve la senda que nunca
se ha de volver a pisar.
Caminante, no hay camino,
sino estelas en la mar.”

Antonio Machado

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ABSTRACT

This master's thesis describes the integration of the driverless taxis in different markets (dispatching, hailing, stand, dispatching-hailing and stand-hailing) and with an exhaustive analysis from a mathematical model how they behave. To carry out this analysis, prior taxi mathematical models for conventional taxis powered by an internal combustion engine (ICE) has been taken as a starting point, as well as integrated into an only one model and converted them to electric vehicles which will be the basis for the driverless taxis.

The way to measure the feasibility of this automated taxis will be by using the system cost which will be composed by the user, the taxi (in contrast to other authors it will not be called the driver cost since it is automated), the infrastructure (needed to provide electricity to the vehicles) and the externality for the city costs. Additionally, this integrated model allows to distinguish between different sorts of distance (either in-vehicle, idle, driving to a charging station or dispatching), as well as their times and velocities associated.

The analysis reveals which market delivers the desired performance in a specific scenario through the optimal system unitary cost and allows to obtain the taxi supply per hour and area of service what by means of the Little formula, lets to know the total number of vehicles needed in a specific city area and trip demand per hour and area of service, just as other parameters. It is found that the best taxi market will change depending on the taxi supply per hour and area of service mentioned, as well as other variables as the city area and the taxi demand.

The model set out in this present project is structured firstly with the state of the art, what for the ease of the reader, allows to understand the basis of this master's thesis. Secondly, the model and problem formulation used for the analysis will be explained. In the third line of work, modelling is compute for a real case, as it is the city of Barcelona. Finally, conclusions from results are presented.

Key words: Taxis; Electric; Driverless; Alternative fuel; Charging station Public transportation, Taxi modelling; Transport on demand; Barcelona; Dispatching; Hailing; Stand; Dispatching-hailing; Stand-hailing;

RESUMEN

Esta tesina de máster describe la integración de los taxis autoconducidos en los diferentes mercados (dispatching, hailing, stand, dispatching-hailing and stand-hailing) y con un exhaustivo análisis a partir de un modelo matemático cómo se comportan. Para llevar a cabo este análisis, se han tomado como punto de partida diferentes modelos ya existentes para taxis convencionales que funcionan con combustibles fósiles, así como se han integrado en un único modelo y transformado para vehículos eléctricos los cuales servirán como base para los vehículos autoconducidos.

La forma de medir la viabilidad de estos taxis automáticos es usando el coste del sistema, el cual está compuesto por el coste del usuario, el del taxi (a diferencia de otros autores, no será llamado coste del conductor), la infraestructura (necesaria para proveer de electricidad el vehículo eléctrico) y el coste de externalidad para la ciudad. Asimismo, el modelo integrado permite diferenciar entre los diferentes tipos de distancia (tanto para cuando el pasajero viaja dentro del vehículo, cuando el vehículo no lleva pasajero, cuando el vehículo se desplaza a cargar el vehículo a la respectiva estación de carga o como cuando el taxi es despachado "*dispatched*"), así como los tiempos y velocidades asociados.

El análisis muestra qué mercado del taxi responde al mejor comportamiento en un escenario específico a través del valor del coste unitario óptimo del sistema que permite obtener la cantidad de taxis por hora y área de servicio y que por medio de la Fórmula de Little permite calcular el número de vehículos necesario en un área específico de una ciudad con una demanda de viajes por hora y área de servicio determinados. Además, se encuentra que el mejor tipo de mercado responde directamente a la cantidad de taxis por hora y área de servicio, así como otras variables, como la demanda y el área de la ciudad.

El modelo expuesto en el presente proyecto está estructurado primeramente con el Estado del Arte, que, para la facilidad del lector, permite entender la base de esta tesina de máster. En segundo lugar, se explica el modelo utilizado para el análisis. En tercer lugar, se aplica el modelo para un caso real, la ciudad de Barcelona. Finalmente, se presentan las conclusiones del trabajo.

Palabras clave: Taxis; Eléctricos; Autoconducidos; Alternativas de combustible; Estaciones de carga; Transporte público, Modelización de taxis; Demanda del transporte; Barcelona; Dispatching; Hailing; Stand; Dispatching-hailing; Stand-hailing;

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List of output variables

VARIABLE	UNIT	DEFINITION
Z	h	Cost of the system
Z_u	h	User cost
Z_u^{trip}	h	Cost of one trip for just one user in a specific area and one hour
Z_t	h	Taxi cost
Z_t^{trip}	h	One trip cost for one taxi in one hour and in a specific area
$Z_t^{all\ trips}$	h	All trips cost for one taxi in one hour and in a specific area
Z_I	h	Infrastructure cost
Z_c	h	Cost for the city
z_u	h	Unitary user cost
z_t	h	Unitary taxi cost
z_i	h	Unitary infrastructure cost
z_c	h	Unitary cost for the city
$Z_{I,Y}$	h	Fixed charging station cost per site for the BEV
$Z_{I,C}$	h	Variable charging station cost factor for the BEV
M	taxis	Number of taxis in the fleet
M_Q	Taxis	Number of taxis servicing
M_E	Taxis	Number of empty taxis
M_C	Taxis	Number of taxis in the charging state
μ	-	Rate of taxis per hour
R	Taxis/h	Number of taxis in the charging state per hour
T	h	Total taxi cycle time
T_W	h	Waiting time of the user
T_A	h	Access time of the user
ΔT	h	Increase in the travel time of the other drivers
T_E	h	Total time in the empty state
T_Q	h	Total time servicing a trip
T_C	h	Total time in the charging state
d	km	Total cycle distance
d_Q	km	Distance servicing a trip
d_D	km	Distance for dispatching to a customer
d_I	km	Distance in the empty state for trip
d_S	km	Distance of the taxi going back to the stand after a service
d_C	km	Distance of the taxi while driving to a charging station
v_Q	km/h	Velocity of the taxis servicing a trip
v_I	km/h	Velocity of the taxis in the idle state
v_D	km/h	Velocity of the taxis for dispatching to a customer
v_S	km/h	Velocity of the taxi going back to the stand after a service

v_C	km/h	Velocity of the taxi in the charging state
v_u	km/h	Velocity of the customer heading to a stand
$\bar{v}$	km/h	Average speed of the taxi travelled by taxis
v_1	km/h	Average speed without the presence of taxis
λ_d	taxis/h·A	Taxi hourly supply per hour and area of service
λ_u	trips/h·A	Trip demand per hour and area of service
$\bar{n}$	trips	Average number of trips per hour and vehicle
VoT	€/min	Value of time of the users
C_h	€/h	Hourly cost of the taxis
C_{km}	€/km	Operational cost per unit of distance of the taxis
C_s	€/stands	Hourly cost of each taxi stand
A	km^2	Area of the city
r	-	Area network parameter depending on the geometry
α_A	-	Customer perception factor of the access time
α_W	-	Customer perception factor of the waiting time
α_Q	-	Customer perception factor of the in-vehicle travel time
α	veh/A·km	Slope of the speed-density linear relation of the macroscopic diagram function
C_E	€/kg of CO_2	Emission unitary cost for all vehicles
E_d	-	Fuel consumption
F_C	-	Emission of various pollutants
s	stands	Number of stands
a	km^2	Area of influence of a stand
Y	sites	Number of charging sites
C_Y	h/site	Fixed charging station cost factor
C_{Y0j}	€/site·h	station installation fixed cost
C_{Y1j}	€/site·h	station maintenance fixed cost
C	ports	Number of charging ports
C_C	h/port	Fixed charging station cost factor
C_{C0j}	€/port·km	Station installation variable port cost
C_{C1j}	€/port·km	Station maintenance variable port cost
C_{Qj}	€/veh·km	Fuel cost
C_{Qm}	€/veh·km	Fleet maintenance cost
C_{M1}	€/veh·km	Agency operating cost per time
C_{M0}	€/veh·km	Fleet depreciation cost
b_1	h	Time spent boarding to the taxi
b_2	h	Time spent alighting to the taxi
b_3	h	Time spent at a charging that is met through hails
b_D	h	Time spend by the customer asking for dispatching a taxi
Q	Ah	Battery capacity
I	A	Load current alternator
$\bar{c}$	€	Average trip cost
D	€	Flag-drop charge
τ_{km}	€/km	Taxi fee per unit of distance
τ_h	€/h	Taxi fee per unit of time
p	h	Time parked at the traffic lights
m	€	Miscellaneous costs
N	trips	Number of trips servicing between charging
TA	km	Total distance driven by an electric taxi without charging

Q	Ah	Battery capacity
I	h	Load current alternator

CHAPTER 1

INTRODUCTION AND OBJECTIVES

1.1. INTRODUCTION

More than 80% of the world's population is expected to live in urban settlements by 2030 (UNFPA, 2007); new alternatives will be required since this change will bring a trade-off to face:

(i) City competitiveness
(ii) Climate change

The **city competitiveness** between the collective (CPT) and the individual (IPT) public transportation modes will be specially challenging. On one side, CPT is featured by being an economy of scale (EOS), where getting larger the size of the transport mode, the average or unit cost gets smaller and when the demand increases as it is expected in the near future, increasing larger fleets becomes cheaper and therefore more competitive. On the other side, IPT will find a larger optimum fleet size since trip demand would increase in a regulated market (administration controls supply and demand), facing the CPT competitiveness. Also, IPT will arise with weightier influence in the congestion and pollution.

The **climate change** has become one of the main problems in the world. Besides, in cities, the biggest factor driving pollution is transport, particularly the private car and in terms of IPT: the taxi. Coming up with this problem to cities is because pollution is a social issue since it affects to people, so it is a problem for the city itself. Particularly, Barcelona has traditionally been a driving force of initiatives to incorporate environmental issues into urban planning and management and to move forward in the application of the principles and values of a culture of sustainability to municipal policies (PECQ, 2011). The Barcelona's City Council launched a plan in 2011 called "The energy, climate change and air quality plan of Barcelona. 2011-2020" aiming the reduction of the increase of greenhouse gas emissions related to Barcelona and the improving of the air quality in the city especially regarding NOx[1] and particulate-matter. The city endured in 2011 709,000 t/year of GHG[2] emissions (an -18% of 2008 value), as well as 2,742, t/year of NOx emissions, 288.1 t/years of PM_{10}[3] and 253.3 and t/year

[1] NOx: nitrogen oxides
[2] GHG: Greenhouse gas
[3] Particulate matter with 10 micrometers or less

of $PM_{2.5}$[4]. Therefore, electric vehicles present itself as a solution to tackle the climate change in the taxi system since there is a reduction in the emissions.

On the other hand, cities have raised several problems with regard IPT as taxis are. From the customers point of view, the taxi industry provides a flexible and fast service. This flexibility comes at a high price per person kilometre, compared to other modes and as a consequence, private taxi markets are mostly local.

Automated taxi vehicles or driverless battery electric vehicles (DBEV) introduces several advantages:

- Cruising velocity might be increased since distance detectors set up in vehicles removed the distance human factor and the security distance between cars may be drastically reduced.
- Since there is no driver, the hourly cost of the taxi may be also decreased and it might imply a reduction of the taxi fare for user, however, it depends on the supply and demand curve and its elasticity: a reduction of the hourly cost might transform into holder license benefit and/or fare reduction, gaining in competitiveness.

There are four sorts of vehicles considered in this study: internal combustion engine vehicle (ICEV), hybrid electric vehicle (HEV), battery electric vehicle (BEV) and the already mentioned DBEV. The ICEV is powered by conventional internal combustion typically by fuels, the HEV is powered by a conventional internal combustion engine and an electric propulsion system that is only powered through the combustion engine, the BEV is powered exclusively by chemical energy (Carpenter, et al., 2013) and the DBEV is powered as a BEV but driven by an automated system integrated in the vehicle. There are also two subcategories for the BEV, the BEV with switching stations (BEVS and DBEVS) that are BEV that refuels in a switching battery station and with recharging station (BEVR and DBEVR), that refuels plug in electricity to the vehicle.

Various taxi markets are considered to simulate: dispatching (D), hailing(H), stand (S), dispatching-hailing (D-H) and stand-hailing (S-H). In the dispatching market, taxis circulate or wait in taxi stands or just parked in a point that fulfils the hypothesis of heterogeneous supply and demand waiting for a call, joining virtual queues managed centrally (dispatching centers); the customers call the operator or use an app requesting for taxi services and the nearest available taxi in the zone (respecting the queue) is assigned to the customer. In the hailing mode, taxis circulate empty searching for a customer, and customers are looking for a taxi in the nearest location to their origin. In the stand market, customers head to a taxi stand where a FIFO (first-in-first-out) system applies for both the customers' and the drivers' queue (Salanova, et al., 2015). In the dispatching-hailing market, taxis can be dispatched while they are idling or parked, but also hailed on their way idling to park. In this last case, taxis would have been hailed before dispatched. In addition, we will consider the stand-hailing market, where taxis can be hailed on their way to the stand.

Some models have been developed so far. Salanova (2015) presents a model that lets to compare the system cost of different agents and define the optimum operational mode for each type of city from a theoretical point of view, which is achieved by developing a new ICEV taxi model based on the generalized cost that takes into account the user, the driver (or taxi in the present study), the infrastructure and the cost for the city for three operational markets: dispatching, hailing and markets. Sathaye (2014) presents an optimization framework for the design of alternative fuel taxi systems and an assessment of optimal costs in a dispatching-hailing taxi service that can be separated either in dispatching or hailing market associated with various fueling options as: ICEV, HEV and BEV.

[4] Particulate matter with 2.5 micrometers or less

The optimization framework provides a basis for solving larger real-world electric taxi systems design problems in the future.

The main contributions obtained in this study are:

- The proposed model is based on previous aggregated mathematical models used for estimating the key performance indicators (KPI): user, taxi, infrastructure and city costs integrated in a generalized cost as (Salanova, et al., 2015) and (Sathaye, 2014) set out for comparing different levels of trip demand and taxi supply per hour and area of service if it is required between markets.
- This model goes a step further and integrates the dispatching-hailing market in a system that allows for comparing with the other markets, as well as the stand-hailing market, that represents a scenario of taxis that after servicing a trip in the stand mode they are heading back for a stand and might be hailed by a customer.
- The introduction of the DBEV conceptually and applied in the model.
- The case study of a city like Barcelona for all the new categories: dispatching-hailing, stand-hailing, DBEVS and DBEVR.

1.2. OBJECTIVES

1.2.1. General objectives

Taking advantage of previous papers with standards vehicles powered by fuel and electricity, the ***general objective*** aims to set out a taxi modelling integrating these studies into one that allows to analyse the behaviour of the forthcoming driverless taxi in different markets: the dispatching, the hailing, the stand, the dispatching-hailing, the stand-hailing.

1.2.2. Specific objectives

In more detail, for securing the attainment of the general goals, it is necessary to set the next ***specific objectives***.

- Literature review on the taxi modelling that comprises the user, taxi, infrastructure and external costs, as well as, takes into account the dispatching, the hailing and the stand markets for standard taxi vehicles (SV) and battery electric taxi vehicles (BEV).
- Selection of the most suited formulation and integration of them in order to set out a new model. Development of the new taxi modelling formulation for the dispatching-hailing and stand-hailing markets and for specifying different distances and associated times and velocities with the intention of studying the performance of the upcoming driverless battery electric vehicles (DBEV).
- Case study in Barcelona: computation and analysis of the developed taxi modelling, variables sensitivity and comparison between different markets and different types of vehicle: SV, BEV and DBEV, with the purpose to predict the behaviour of the foresight integrated DBEV in the taxi fleet size of Barcelona.
- Edition of the results and conclusions related to the accuracy of the taxi modelling simulation.
- Economic influence of the new technologies, as automated vehicles in individual public transportation terms.

1.3. STRUCTURE

This master's thesis is subdivided into five chapters, references and an appendix as it is observed in figure (1)

- **Chapter 1** consists in the introduction of the work, the main objectives, distinguishing between the general and the specific goals.
- **Chapter 2** is composed by the State of the Art providing the basic concepts to understand this study. It includes a brief description of the current situation of taxis and their different markets.
- **Chapter 3** is composed by the Problem Formulation of the mathematical model. It is comprised by the general equations for each cost: the user, the taxi, the infrastructure and the external costs with all the variables and parameters related. This chapter is inextricably linked with the appendix. Specific conclusions of the analysis are presented.
- **Chapter 4** comprised the analysis of the modelling applying for a real case as it is Barcelona.
- **Chapter 5** presents main conclusions and outlooks distinguishing by the general and specific ones. A discussion of the results is done and a guideline is presented for future research.
- The **Appendix** encloses this work going in depth with the cost general equations showed at Chapter 3.

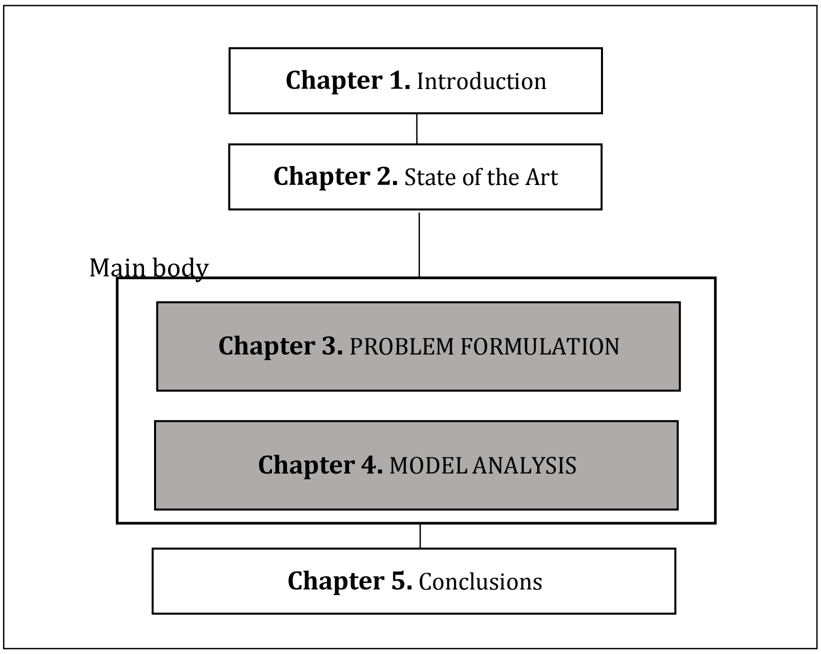


Figure 1. Structure of the master's thesis

CHAPTER 2

STATE OF THE ART

2.1. INTRODUCTION

Chapter two starts introducing the basic of the taxi modelling. First of all, a brief historical description of the modelling is explained, distinguishing two types:

(i) Aggregated and equilibrium models.
(ii) Simulation based models.

Added to that, currently models and formulation is presented, highliting the different types of markets and vehicles, as well as four researches and their main studies related with this master's thesis: Salanova (2011), Sathaye (2014), **Schroeder** (2012) and Lidicker (2011).

2.2. HISTORICAL DESCRIPTION OF THE TAXI MODELLING

2.2.1. Introduction

2.2.2. Aggregated and equilibrium models

Since 1972, different authors have carried out different researches developing some aggregated and equilibrium models. The timeline of this models as observed in the next table (1) has been complied by Salanova (2015). The socioeconomics and the local spatial characteristics, as well as different agents of the taxi markets, have a significant impact on the performance of the taxi markets. This is the main limitation of these models.

Table 1. Agreggated and equilibrium models chronology

Douglas (1972)	First model that evaluates the performance of the taxi services. Uses economic relationships of the goods and services sectors.
Vany (1975)	Introduces the value of time of customers and the waiting time in the demand assumptions. Uses Douglas demand function as a starting point.
Schroeter (1983)	Uses the dispatching and the airport cabstand as modes of operation.
Manski and Wright (1976)	Operational mode. Introduces the taxi stand.
Cairns and Liston-Heyes (1996)	Operational mode. Uses Douglas demand functions as a starting point and redefines them. They assumed uniform demand within the day, which decreases as waiting time increases.
Arnott (1996)	Operational mode. Analyses the shadow cost of taxis in the first best solution (minimum cost), proposing subsidization for covering these costs in the vacant trips of taxis.
Yang and Wong (1997)	First equilibrium model. Takes into account the spatial distribution of demand and supply in the city using traffic assignment models.
Yang et al. (1998, 2000, 2005 and 2010)	Adds the possibility to take into account the spatial distribution of demand and supply in the city into account.
Salanova (2011)	A detailed review of the aggregated and equilibrium models of taxi services.

2.2.3. Simulation based models

Since 1987, different authors developed some simulation based models. The timeline of this models as observed in the next table (2) has been complied by Salanova (2015) along with the aggregated and equilibrium models.

Table 2. Simulaled based models chronology

Bailey and Clark (1987)	The first simulation model. Concludes that the waiting time is relatively insensitive to changes in demand but highly sensitive to changes in the number of taxis.
Bailey and Clark (1992)	Simulate dispatching taxi market. Concludes that there is a linear relation between the total traveled distance and the fleet size.
Kim et al. (2005)	Developes a simulation-based model for taxi stand services and proved that the use of information technologies can improve the quality of taxi services by 20% using a simulation-based stand taxi services model.
Song and Tong (2006) and later Tong (2006)	Simulates the taxi stand market and highlights the limitations of the aggregated models such as the time-dependent patterns or the non-equilibrium in the regulated taxi markets.
Lioris et al. (2010)	Developes a discrete-event simulation model for reproducing real-world taxi on demand market conditions
Salanova et al. (2013)	Presents an agent-based model for simulating taxi services, including the three operation modes: dispatching, hailing and stand.
Sathaye (2014)	Introduces Battery Electric Vehicles and takes into account the dispatching and hailing markets.

2.3. CURRENT FORMULATION PRESENTED IN THE LITERATURE

2.3.1. Types of vehicles considered

In this section, different types of vehicles and their characteristics are explained.

Internal combustion engine vehicle (ICEV). These kinds of vehicles are powered, as their name says, by a conventional internal combustion engine, typically using fuels like diesel or gasoline. Due to this vehicle is provided by a tank fuel large enough to store the amount of fuel necessary to complete one shift, the range will be not considered in this type of vehicles.

Micro hybrid. For these vehicles, the electricity is just used for start/stop function.

Mild hybrid. The electric motor works with the combustion engine, however, it is not possible to work exclusively with electricity.

Hybrid-electric vehicles (HEV). These vehicles are powered by a conventional internal combustion engine and an electric propulsion system. However, it is not possible to charge the electric propulsion system from an external source and must be charged through the energy obtained with the combustion engine. Since HEV can have a greater range than ICEV and ICEV is not limited, we assume in this study that this kind of model is not limited by range.

Standard vehicles (SV). These sorts of vehicles bring together the ICEV and HEV, which use petroleum. It will be useful to simplify these two types of vehicles hereinafter.

Plug-in hybrid electric vehicles (PHEV). In this case, batteries can be full restored by plugging an external power source. Since this kind of vehicle model works as an HEV with the only difference that the battery can be charged by an external source, range is not limited. For this sort of vehicle, once batteries are depleted, the combustion engine works to propel the vehicle until the end of the trip while battery just provide power to the electronic on-board. Also, PHEV will not be refuelled more often than the conventional cars, as the main advantage of this type of vehicle is to reduce the fuel use and its cost by power the vehicle with electricity.

Hybrid with range extender (RXBEV). They behave like BEV plus by means of a combustion engine that produces electricity, the range is extended.

Battery electric vehicles (BEV). This sort of model is powered exclusively by chemical energy stored in a rechargeable battery. The way to charge the battery is connecting by plugging an external power source, although a new full battery can replace the depleted battery and we will consider this way for the present work since switching batteries takes much shorter time, due to the switching process will be around 80 seconds. Therefore, this type of vehicles will need a switching station that will allow to switch batteries in BEV as quickly as the vehicles powered by conventional fuel (Carpenter, et al., 2013). Besides, these vehicles will be distinguished in the ones that switch their battery for a new one using swapping station: **battery electric vehicles with swapping stations (BEVS)**, and the ones which need to plug-in to recharge the integrated battery in recharging stations: **battery electric vehicles with recharging stations (BEVR)**.

2.3.2. Types of markets involved

There are 5 types of markets: **dispatching (D)**, **hailing (H)**, **stand (S)**, **dispatching-hailing (D-H)** and **stand-hailing (S-H)**. In the **dispatching** market, taxis circulate or wait in taxi stands or just parked in a point that fulfils the hypothesis of heterogeneous supply and demand waiting for a call, joining virtual queues managed centrally (dispatching centers); the customers call the operator or use an app requesting for taxi services and the nearest available taxi in the zone (respecting the queue) is assigned to the customer. In the **hailing** mode, taxis circulate empty searching for a customer, and customers are looking for a taxi in the nearest location to their origin. In the **stand** market, customers head to a taxi stand where a FIFO (first-in-first-out) system applies for both the customers' and the drivers' queue (Salanova, et al., 2015). In the **dispatching-hailing** market, taxis can be dispatched meanwhile they are idling or parked, but also hailed on their way idling to park. In this last case, taxis would have been hailed before dispatched. In addition, we will consider the **stand-hailing** market, where taxis can be hailed on their way to the stand.

The main advantage of the **stand** market is that the distance circulated without a customer is drastically reduced since taxis do not circulate looking for a customer; on the other hand, the disadvantage is that customers must access the taxi stands for getting served. The **dispatching** market has the advantage that drivers are not randomly looking for a customer, reducing the waiting time, especially in non-peak hours, where the demand is lower, but operational costs are increased since there is a need for a dispatching center. Later, the **hailing** market presents a priori larger vacant distance and congestion impacts on the network (Salanova, et al., 2015) and also we will consider that they will circulate without stopping during the whole day, meanwhile either in the dispatching they might be parked and in the stand taxis will be stopped in a stand waiting for the customer. It is important to notice that for the hailing market with the BEV types, the pollution generated is drastically reduced both for taxis and other drivers. In terms of the **dispatching-hailing** and **stand-hailing**, the main advantage will be the introduction of a vacant scenario where taxis can be hailed in their way to being back to their stand for being dispatched or waiting for a passenger. Although there has been relatively little research on both the dispatching-hailing and stand-hailing market they will have the advantages and disadvantages mentioned before, always related with the proportion of the types of market demand.

This study highlights the hailing mode for DBEV. It is important to mention it is the human factor that detects a customer hailing in the street and there is so far, no prior technology research in this way. We will consider this possibility even though up until now it is not possible to integrate this system in the real market. For the sake of the ease understanding we will state that it is a fictitious market.

2.3.2.1. Dispatching market

The first part of the figure (3) corresponds to the dispatching market. In this state, taxis will have two states: servicing; either dispatched or servicing a trip, and empty. Taxis are dispatched-assigned when they are in the empty mode what means they might be either parked or just idling. Once they are assigned and vehicles get to the customer, the state mode turns into in-service till the end of the trip with the customers. Once the taxi drops off the passenger, taxi turns into the empty state again.

2.3.2.2. Hailing market

The second part of the figure (3) represents the hailing market. They circulate without stopping until a customer hails them. Therefore, this mode will have to states: empty, where taxis will circulate hailing for a customer right after the drop-off, and in-service. For BEV, charging state is added

2.3.2.3. Stand market

As observed in the third part in figure (3), the stand market has two states: empty; where taxis wait in a stand and also when they drop off the customer and ride to the stand, and in-service as a second state. When the stand market works with BEV, taxis need to be charged.

2.3.2.4. Dispatching-hailing market

In the dispatching-hailing market have two states: in-service mode, where taxis are dispatched, as well as, servicing a trip, and the empty mode, once taxis drop off customers they head to the be parked to wait for the next call, although they can be hailed on their way to be parked figure (2 and 3).

Prior investigations have proposed the introduction of the parameter γ as the proportion of trips hailed as (Sathaye, 2014). This study establishes the hailing proportion of demand with γ and with $(1-\gamma)$ for the dispatching one.

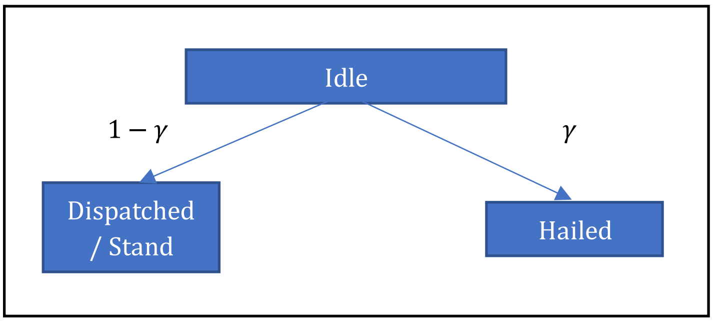
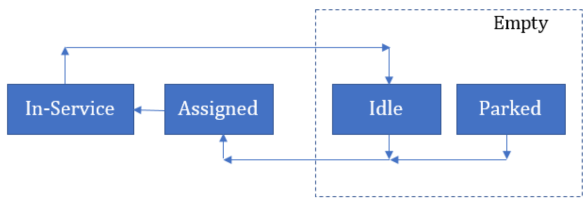
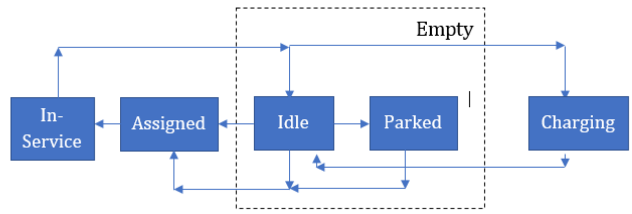
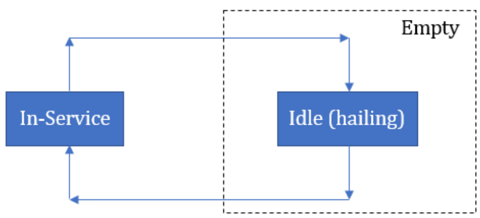
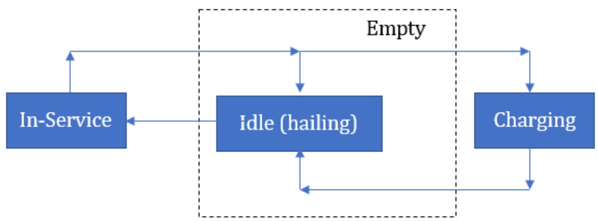
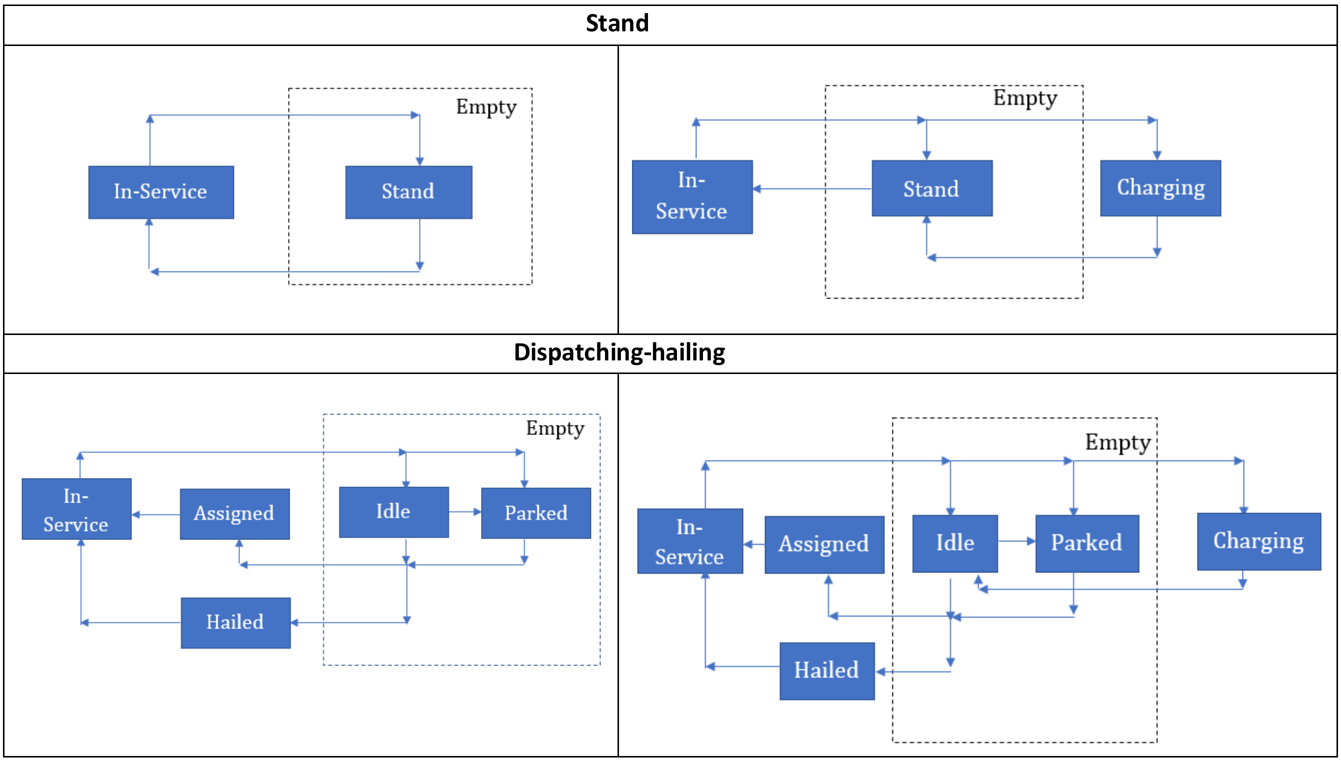


Figure 2. Distribution of dispatched/stand and hailed vehicles after idling

Table 3. SV and BEV taxi system markets

Standard vehicles	**Battery electric vehicles**
Dispatching	
	
Hailing	
	



Once the battery vehicle is about to be depleted after driving N trips, taxis will must go to the charging station to either be recharged by plug-in or swapping the battery. Hence, taxis need to be charged after servicing N trips. For all the BEV markets, if N is the number of trips serviced between charging, taxis transfer directly from in-service to empty after $[N-1]/N$ fraction of drop-offs, or to charging after $[1/N]$ fraction of drop-offs (Sathaye, 2014).

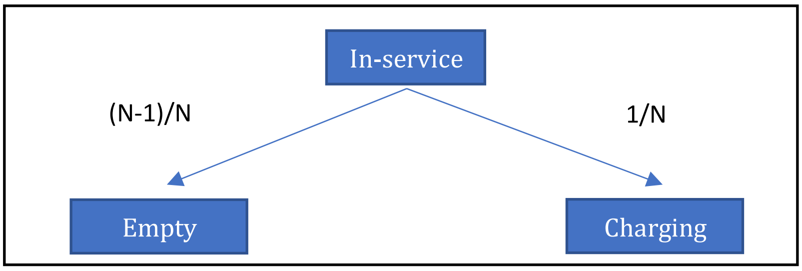


Figure 3. Distribution of empty and charging states after servicing a trip

2.3.3. Types of vehicles considered

In this section, different types of vehicles and their characteristics are explained.

Internal combustion engine vehicle (ICEV). These kinds of vehicles are powered, as their name says, by a conventional internal combustion engine, typically using fuels like diesel or gasoline. Due to this vehicle is provided by a tank fuel large enough to store the amount of fuel necessary to complete one shift, the range will be not considered in this type of vehicles.

Micro hybrid. For these vehicles, the electricity is just used for start/stop function.

Mild hybrid. The electric motor works with the combustion engine, however, it is not possible to work exclusively with electricity.

Hybrid-electric vehicles (HEV). These vehicles are powered by a conventional internal combustion engine and an electric propulsion system. However, it is not possible to charge the electric propulsion system from an external source and must be charged through the energy obtained with the combustion engine. Since HEV can have a greater range than ICEV and ICEV is not limited, we assume in this study that this kind of model is not limited by range.

Standard vehicles (SV). These sorts of vehicles bring together the ICEV and HEV, which use petroleum. It will be useful to simplify these two types of vehicles hereinafter.

Plug-in hybrid electric vehicles (PHEV). In this case, batteries can be full restored by plugging an external power source. Since this kind of vehicle model works as an HEV with the only difference that the battery can be charged by an external source, range is not limited. For this sort of vehicle, once batteries are depleted, the combustion engine works to propel the vehicle until the end of the trip while battery just provide power to the electronic on-board. Also, PHEV will not be refuelled more often than the conventional cars, as the main advantage of this type of vehicle is to reduce the fuel use and its cost by power the vehicle with electricity.

Hybrid with range extender (RXBEV). They behave like BEV plus by means of a combustion engine that produces electricity, the range is extended.

Battery electric vehicles (BEV). This sort of model is powered exclusively by chemical energy stored in a rechargeable battery. The way to charge the battery is connecting by plugging an external power source, although a new full battery can replace the depleted battery and we will consider this way for the present work since switching batteries takes much shorter time, due to the switching process will be around 80 seconds. Therefore, this type of vehicles will need a switching station that will allow to switch batteries in BEV as quickly as the vehicles powered by conventional fuel (Carpenter, et al., 2013). Besides, these vehicles will be distinguished in the ones that switch their battery for a new one using swapping station: **battery electric vehicles with swapping stations (BEVS)**, and the ones which need to plug-in to recharge the integrated battery in recharging stations: **battery electric vehicles with recharging stations (BEVR)**.

2.3.4. Aggregated problem formulation review

Several authors have done intense studies regarding taxi modelling focusing on aggregated based on continuous variables. Various models have been developed so far for assessing the costs in terms of different agents involved as the user, the driver, the infrastructure and the external cost for the city, as well as the waiting and access time for the customers and income of taxi drivers through fares, providing policy makers or decision variables with methodologies for estimating the optimum fleet size for each demand level and city parameters like geometry and its congestion levels as well as identifying the best market for each city and taxi supply.

Salanova (2015) introduces a model able to compare the costs of the different actors and define the optimum operational mode for each type of city from a theoretical point of view, which is achieved by developing a new taxi model based on the generalized cost. The proposed model uses the different mathematical formulations presented in the literature for estimating the optimum fleet size related to each operational mode, city size and demand level. The correspondent generalized cost and waiting/access time of the customers are also obtained, comparing the performance of the three operational modes for the same city. These results can be used by the policy makers in order to define the taxi operation mode for the different areas of the city and time intervals of the day, which even if it may be a combination of various modes, it can favor the one having the smaller system unitary cost.

This model proposed by Salanova (2015) is the basis of the present work, and proposes an objective function (), with the fleet size, λ_d, being the decision variable. The general function is the sum of the driver, Z_d, user, Z_u, external, Z_C, and infrastructure costs, G. Besides, he applies this model for conventional vehicles in the dispatching, hailing and the stand markets.

$$Min\, Z = Z_d + Z_u + Z_c + G \tag{2. 1}$$

$$Z_u = \cdot \left[\alpha_A \cdot T_A + \alpha_W \cdot T_W + \alpha_{IV} \cdot T_{IV} + \frac{\bar{c}}{VoT}\right] \tag{2. 2}$$

$$Z_d = \frac{\lambda_d}{\lambda_u VoT}\left[-\bar{n} \cdot \bar{c} + \left(\bar{n} \cdot \bar{d} \cdot C_{km} + C_h\right)\right] \tag{2. 3}$$

$$Z_c = \lambda_v \cdot \frac{\Delta T_v \cdot VoT_v}{\lambda_u VoT} + \frac{\lambda_d \cdot C_E \cdot E_d}{\lambda_u VoT} + \frac{\lambda_v \cdot C_E \cdot \Delta T_v \cdot E_d}{\lambda_u VoT} \tag{2. 4}$$

On the other hand, **Sathaye (2014)** develops a model based on transit systems design methods and focuses on developing an approximate analytic model for electric taxi systems (SV, PHEV and BEV), to address large-scale taxi systems design problems. He proposes two objective function in terms of the type of vehicle,

(i) For ICEV, HEV and PHEV

Agency costs are made up of costs associated with the fleet, which are based on distance traveled per cycle $\{d_Q + [1-\gamma]d_D + d_I\}$, operating time $\{\omega_M \cdot M\}$, and infrastructure cost $\{\omega_Y \cdot Y + \omega_C \cdot M\}$ for PHEVs. For ICEVs and HEVs, $\{\omega_Y \cdot Y + \omega_C \cdot M\}$ can be set to 0, as this only pertains to PHEVs. $\{\omega_Y \cdot Y + \omega_C \cdot M\}$ is comprised of a fixed charging station site cost $\{\omega_Y \cdot Y\}$, and a variable cost per port $\{\omega_C \cdot M\}$ for charging PHEVs. User costs are comprised of a cost associated with travel $\{d_Q/\, v\}$ and a cost for waiting for the nearest empty taxi $\{d_D/\, v\}$ (Sathaye, 2014).

$$Z_d = \left\{\omega_Q \cdot M \cdot \frac{d_Q + [1-\gamma]d_D + d_I}{T} + \omega_M \cdot M + \omega_Y \cdot Y + \omega_C \cdot M\right\} + \left\{\frac{d_Q}{v} + \frac{d_D}{v}\right\} \tag{2. 5}$$

$$M = M_Q + M_E \tag{2. 6}$$

$$T = \left[b_1 + b_2 + \frac{d_Q}{v} + [1-\gamma]\frac{d_D}{v}\right] + \left[\frac{d_I}{v_I}\right] \tag{2. 7}$$

(ii) For BEV

Agency costs are made up of costs associated with the fleet and infrastructure costs. $\{M_C \cdot b_3 / \left[b_3 + \frac{dc}{v}\right]\}$ is the number of taxis at charging stations (Sathaye, 2014).

$$Z_d = \left\{\omega_Q \cdot M \cdot \frac{d}{T} + \omega_M \cdot M + \omega_Y \cdot Y + \omega_C \cdot C\right\} + \left\{\frac{d_Q}{v} + \frac{d_D}{v}\right\} \tag{2. 8}$$

$$C = M_C \cdot b_3 \Big/ \left[b_3 + \frac{d_C}{v}\right] \tag{2. 9}$$

$$M = M_Q + M_E + M_C \tag{2. 10}$$

$$T = \left[b_1 + b_2 + \frac{d_Q}{v} + [1-\gamma]\frac{d_D}{v}\right] + N\left[\frac{d_C}{N \cdot v} + \frac{d_I}{v_I}\right] + \left[b_3 + \frac{d_c}{v}\right] \tag{2. 11}$$

$$d = N\left[d_Q + [1-\gamma]d_D + d_I\right] + 2 \cdot d_C \tag{2. 12}$$

Apart from these two objective functions proposed, **Sathaye (2014)**, considers three different cases for how empty taxis are located within a station influence area, which correspond to different potential values for the fleet size.

(i) High values for the fleet size. Available taxis are uniformly distributed (4. a).
(ii) Intermediate values for the fleet size. The density of available taxis decreases with distance from stations, but is uniform beyond some point r_1 (4. b).
(iii) Low values for the fleet size. The density of available taxis is decreasing (4. c).

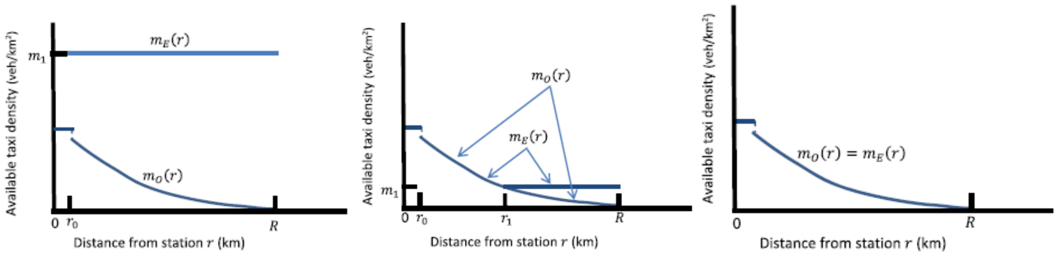


Figure 4. Spatial distribution of available taxis for: a) which r1=0 and m1>0, b) which 0<=r1<=R, c) which r1=R and m1=0 (Sathaye, 2014)

By way of the infrastructure cost for charging stations, this State of the Art distinguishes two types of charging stations: recharging and swapping stations:

(i) **Schroeder (2012)** studies the **recharging stations** by plug-in. In his research to obtain the estimated Return on Investment (ROI) of fast charging infrastructure for electric vehicles distinguishes the infrastructure cost between the CAPEX and OPEX, what leads to study deeply where the costs come from and allows to set out studies to reduce the cost.

(ii) **Lidicker (2011)** proposes and strategy for charging electric vehicles by switching batteries in **swapping station** in a leasing market. One of the main advantages of this market is the time switching this battery: 8 seconds

CHAPTER 3

PROBLEM FORMULATION

3.1. INTRODUCTION

The main objective of this work is to compare the different vehicle types in the next markets: dispatching, hailing, stand, dispatching-hailing and stand-hailing. One of the parameter chosen for this comparison is the taxi fleet size needed, M, or the taxi hourly supply per area, λ_d, both terms related by the Little's equilibrium formula. Besides, the unitary cost is required since it is strongly associated with the fleet size.

In order to find the values required a general formulation is defined and later individualized for every type of vehicle and market. The focus of the problem formulation is to be able to define multiple variables and its relation in an overall idea. This is useful since the less specific is the formulation the more analyze is able to obtain, especially for future studies.

The methodology carried out in this chapter is as follows:

(i) **Assumptions**. In this part are defined the main assumptions of the problem formulation as the types of markets (dispatching, hailing, stand, dispatching-hailing, stand-hailing) and vehicles (SV, BEV and DBEV) considered, among other important considerations like the behavior of variables and parameters.

(ii) **Background.** Several variables take place in the main equations involved in the general formulation, they are explained.

(iii) **General formulation.** The objective function and its main equations involved are described, as well as, the most important variables to be optimized.

(iv) **Fleet size.** As explained in following sections, the objective function has as optimized variable the taxi supply per hour and area of service, λ_d. This variable must be converted into the total number of vehicles or fleet size, M.

(v) **Appendix.** Each type of market has its own final equations because of their particularities. They are accurately described for each type of vehicle and market.

3.2. ASSUMPTIONS

(i) In this study are considered five types of market:
   a. Dispatching based on Salanova (2015).
   b. Hailing based on Salanova (2015).
   c. Stand based on Salanova (2015).
   d. Dispatching-hailing based on Sathaye (2014).
   e. Stand-hailing. It is one of the main contributions of this study. In this market, taxis will have two states: in-service mode, where taxis are servicing a trip, and the empty mode, once taxis drop off customers they head to a stand in order to wait for a customer who has to get to the stand. Meanwhile taxis are heading for a stand they might be also be hailed by a customer (5). In the BEV and DBEV case, taxis will have added the charging state

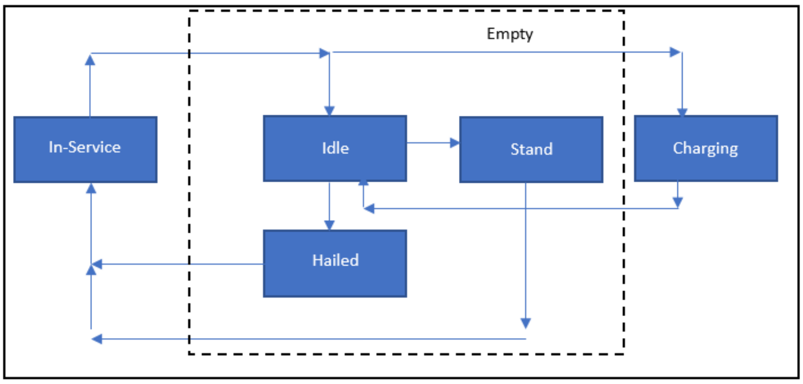


Figure 5. Stand-hailing taxi market

(ii) The special features of the different markets are summed up in the next table,

Table 4. Features of the different markets

	D	H	S	D-H	S-H
Times	$T_a = 0$	$T_a = 0$	$T_W = 0$	$T_a = 0$	-
Distances for SV	d_D, d_Q, d_I	d_Q, d_I	d_S, d_Q	d_D, d_Q, d_I	d_S, d_Q, d_I
Distances for BEV and DBEV	d_D, d_Q, d_I, d_C	d_Q, d_I, d_C	d_S, d_Q, d_C	d_D, d_Q, d_I, d_C	d_S, d_Q, d_I, d_C

(iii) There will be considered that taxis at the hailing market will be running during all the time.

(iv) There will be always a taxi waiting at the stand for the next passenger in the stand and stand-hailing markets.

(v) In the dispatching market, the time that takes a customer to order a taxi will be zero.

(vi) It is assumed that there is just one taxi operator.

(vii) This model takes into account three types of vehicles:
   a. SV based on Salanova (2015).
   b. BEV based on Sathaye (2014).
   c. Automated electric vehicles or driverless battery electric vehicles (DBEV). These vehicles will behave as BEV but will have as a main distinguishing feature that they will me automated and there will be no driver. This will not bring a change in the formulation as happens between SV and BEV, consequently in such vehicles,

integrated in the BEV formulation, will have a reduction in the hourly cost and an increasing in the velocity, as it will be seen below. They will also follow the swapping and recharging criteria: **driverless battery electric vehicles with swapping stations (DBEVS)** and **driverless battery electric vehicles with recharging stations (DBEVR).** Likewise, it will be also one of the main contributions of this work.

(viii) Either supply or demand are considered spatially uniformly distributed, constant, continuous and deterministic. The demand also is considered inelastic to variations in the cost and the service quality.
(ix) Expected values according to approximate values are used for distance and time.
(x) Velocities remain all over the scenario features since there is no congestion considered.
(xi) Users pick up the nearest empty taxi.
(xii) Each trip is assumed to carry only 1 person.
(xiii) For the BEV and DBEV, taxis fully recharge when depleted battery, the range limitation is reached and there is no remaining power when they get to the recharging/swapping station.
(xiv) For the BEV and DBEV, when taxi fully recharge or switch their battery, they move away from stations in order to create a steady-state density all over the area of service. This model assumes that taxis move outwards from these stations to reach areas with lower densities (it is an ideal scenario, since allows to decrease the waiting time).
(xv) The number of recharging/swapping, M, and stand stations, s, are homogeneously distributed all over the area of service.

3.3. BACKGROUND

3.3.1. In-vehicle travel distance, d_Q

The in-vehicle travelled time will be the **distance travelled by a taxi while servicing a trip**, d_Q, This distance will be the average distance between two random points in a uniformly distributed area for a circular region (Daganzo, 1978),

$$d_Q = 0.51 r\sqrt{A} \tag{3. 1}$$

And for a square area (Daganzo, 1978),

$$d_Q = 0.52 r\sqrt{A} \tag{3. 2}$$

3.3.2. In-vehicle travel time, T_Q

The trip distance is calculated by considering the region as a square of side and estimating the expected distance between two random points within the region. The distance between two random points in a region is equal to the half of the square of the area. The expected travel time is the factor between this expected distance and the average speed (Daganzo, 1978).

$$T_Q = \frac{d_Q}{v_Q} \tag{3. 3}$$

3.3.3. Charging time, b_3

The charging time, b_3, will be the battery capacity in *[Ah]*, Q, divided by the load current alternator, I, in [*h]*

$$b_3 = \frac{Q}{I} \tag{3. 4}$$

3.3.4. The average trip cost or taxi revenue, $\bar{c}$

The **average trip cost** or **taxi revenue,** $\bar{c}$, is the sum of the cost of the flag-drop, D, the cost of the distance travelled by a taxi while serving a trip, $d_Q \cdot \tau_{km}$, the cost of waiting at the traffic lights and the miscellaneous costs, m, (Carpenter, et al., 2013),

$$\bar{c} = D + d_Q \cdot \tau_{km} + p \cdot \tau_h + m \tag{3. 5}$$

3.3.5. Operational cost per unit of distance of taxis, C_{km}

The operational cost per unit of distance of taxis, C_{km}, will be the sum of the fuel cost, C_{Qf} and the fleet maintenance cost, C_{Qm}, (Sathaye, 2014)

$$C_{km} = C_{Qf} + C_{Qm} \tag{3. 6}$$

3.3.6. Hourly operational cost of the moving taxis, C_h

The hourly operational cost of the moving taxis, C_h, will be the sum of the agency operating cost per time, C_{M1} and the fleet depreciation cost, C_{M0}, (Sathaye, 2014),

$$C_h = C_{M1} + C_{M0} \tag{3. 7}$$

3.3.7. Average number of trips per hour and taxi, $\bar{n}$

The average number of trips per hour and taxi, $\bar{n}$, (Salanova, et al., 2015)

$$\bar{n} = \frac{\lambda_u}{\lambda_d} \tag{3. 8}$$

3.3.8. Range or Number of trips servicing between charging, N

In this model it is assumed that the autonomous capability of the battery power bank is shorter than the fuel tank, therefore, there is range limitation. The number of trips servicing between charging for the BEV type will be the total distance driven by an electric taxi without charging, TA, divided by trip distance done by a taxi, d.

$$N = \frac{TA}{d} \tag{3. 9}$$

It is interesting to know that the range of a BEV is about a third of the conventional vehicle. Thus, BEVs must be refueled about three times more often than a conventional one, SVs (Carpenter, et al., 2013). Therefore, we will not consider ranges for vehicles with conventional fuel. We will assume that TA for the BEV will be 400 km.

3.4. GENERAL FORMULATION

3.4.1. Objective function

Assuming there is a city of area A, with a value of time for the users VoT_u, where the hourly cost for taxis is C_{km} and their distance cost is C_{km}, the average fare as explained in the prior section is $\bar{c}$. Besides, in terms of externalities, the fuel consumption represents E_d, the emission of various pollutants F_c, the slope of the speed-density linear relation of the macroscopic diagram function is α and the average speed without the presence of taxis is v_1. The objective function (Z), will be the minimum of the **unitary cost,** Z**,** as observed in function (3.10)

$$Min\, z = \frac{Z_u + Z_t + Z_c + Z_I}{A \cdot \lambda_u} \tag{3. 10}$$

The decision variable in this study will be: the optimum hourly taxi supply per area, λ_t^*, and the fleet size, M, since they are the most important decision variables because they are regulated by the responsible authority in an attempt to reduce the system cost of the taxi services (Salanova, et al., 2015). In order to obtain them, it will be required to minimize the objective function that

$$\frac{\partial z}{\partial \lambda_d} = \frac{1}{A \cdot \lambda_u} \cdot \frac{\partial}{\partial \lambda_d}(Z_u, \quad Z_t, \quad Z_c, \quad Z_I) = 0 \quad \rightarrow \quad \lambda_d^* \quad \rightarrow \quad M \tag{3. 11}$$

3.4.2. User cost, Z_u

The **trip cost for one user in a specific area,** Z_u^{trip}, in **equivalent hours of user** is the sum of the access time, T_A, the waiting time, T_w, the in-vehicle travel time, T_Q, and the fare or average trip cost expressed in **time units** (Salanova, et al., 2015)

$$Z_u^{trip}[h/trip] = T_A + T_w + T_Q + \frac{\bar{c}}{VoT} \tag{3. 12}$$

This cost can be expressed in **monetary units**,

$$Z_u^{trip}[€/trip] = VoT \cdot (T_A + T_w + T_Q) + \bar{c} \tag{3. 13}$$

The **all trips cost for all the users in a specific area** is obtained multiplying the cost of one trip for just one user in a specific area and one hour by the hourly demand for taxi trips and the area of the region (Salanova, et al., 2015),

$$Z_u = \lambda_u \cdot A \cdot Z_u^{trip} \tag{3. 14}$$

3.4.3. Taxi cost, Z_t

The **one trip cost for one taxi in one hour and in a specific area of service**, Z_d^{trip}, in **monetary units** is the sum of the operational cost of the distance, $d \cdot C_{km}$, and the hourly operational cost of the moving taxis, $\frac{\lambda_d}{\lambda_u} \cdot C_h$, minus the trip fare income, $\bar{c}$ (Salanova, et al., 2015)

$$Z_t^{trip}[€/trip] = -\bar{c} + d \cdot C_{km} + \frac{\lambda_d}{\lambda_u} \cdot C_h \tag{3. 15}$$

For the ease understanding, it can be seen how this expression above behaves like an income (I) /cost (C) equation, and the benefit (B) of the taxis is expressed in negative values.

$$Z_t^{trip}[€] = -I + C = -B \tag{3. 16}$$

This cost can be expressed in **equivalent hours of the user**,

$$Z_t^{trip}[h/trip] = \frac{1}{VoT}\left[-\bar{c} + d \cdot C_{km} + \frac{\lambda_d}{\lambda_u} \cdot C_h\right] \tag{3. 17}$$

The **all trips cost for one taxi in one hour and in a specific area**, $Z_d^{all\ trips}$, in **monetary units** can be obtained multiplying Z_t^{trip} for $\frac{\lambda_u}{\lambda_d}$ that represents

$$Z_t^{all\ trips}[€] = \frac{\lambda_u}{\lambda_d}[-\bar{c} + d \cdot C_{km}] + C_h \tag{3. 18}$$

And in **equivalent hours of the user**,

$$Z_t^{all\ trips}[h] = \frac{\lambda_u}{\lambda_d VoT}[-\bar{c} + d \cdot C_{km}] + \frac{C_h}{VoT} \tag{3. 19}$$

The **all trips cost for all taxis in one hour and in specific area** is the all trips cost for one taxi in one hour and in a specific area multiplied by the taxi hourly supply, λ_d, and the area, A, (Salanova, et al., 2015),

$$Z_t\ [€] = \lambda_u \cdot A[-\bar{c} + d \cdot C_{km}] + \lambda_d \cdot A \cdot C_h \tag{3. 20}$$

And in **equivalent hours of the user**,

$$Z_t\,[h] = \frac{\lambda_u \cdot A}{VoT}[-\bar{c} + d \cdot C_{km}] + \frac{\lambda_d \cdot A \cdot C_h}{VoT} \tag{3. 21}$$

3.4.4. External cost for the cities, Z_c

For **the external cost for the cities,** since this present work is done to compare electric with conventional fuel cars, taking into account the cost for the city (Z_C) will be necessary. This cost will be set by the total cost for the vehicles circulating around the city due to the increase of the density because of the taxis, the pollution that taxis produce while running and the pollution made by the other taxis circulating around city due to the extra time circulating as a result of the taxis (Salanova, et al., 2015).

Either for the dispatching, the hailing or the stand market, the external cost for the city will be taken into account, even though another works did not as (Salanova, et al., 2015) since this cost affects more for hailing than for the others because taxis keep running the whole hour,

$$Z_C = \lambda_u \cdot A\left[\frac{\alpha\lambda_d}{v_1}\left(1 + \frac{C_E E_d}{VoT} + \frac{F_C}{VoT}\right)\right] + \frac{\lambda_d \cdot A \cdot C_E E_d}{VoT} \tag{3. 22}$$

3.4.5. Infrastructure cost, Z_I

There will be taken into account the **stand infrastructure** for all vehicles in the stand market and the recharging and **swapping station** in all markets for BEV and DBEV. These infrastructures there will be related with a cost.

3.4.5.1. Infrastructure cost of the stands for SV, BEV and DBEV

The **infrastructure cost** related with the stand infrastructure, will be (Salanova, et al., 2015),

$$Z_I = \frac{C_s \cdot s}{VoT} \tag{3. 23}$$

Where C_s is the cost of each taxi stand and s the number of stands. Each stand serves and area of a^2 and therefore, the number of stands will be A/a^2 (6). For this model, it is assumed that there will be always one taxi in each stand waiting for a customer.

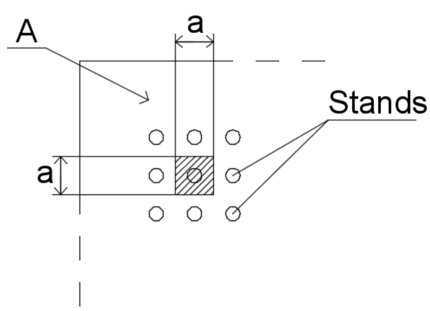


Figure 6. Infrastructure stand emplacement

$$s = \frac{A}{a^2} = A \cdot (\lambda_d - \lambda_u \cdot \bar{t}) \tag{3. 24}$$

3.4.5.2. Infrastructure cost of the battery swapping and recharging by plug-in stations for BEV and DBEV

The infrastructure cost of the BEV and DBEV types will have two sorts of stations: the swapping station, where depleted batteries will be replaced for a new ones (Lidicker, et al., 2011) and the recharging stations (Schroeder, et al., 2012). These two different types will be distinguished by having different fixed cost per site, C_Y, and variable cost per port, C_c (Sathaye, 2014).

The fixed charging station cost per site for the BEV and DBEV (Sathaye, 2014),

$$Z_{I,Y} = C_Y \cdot Y \tag{3. 25}$$

Where C_Y is the fixed cost per site, what is the sum of the station installation fixed cost, C_{Y0j}, and the station maintenance fixed site cost, C_{Y1j},

$$C_Y = \frac{C_{Y0j} + C_{Y1j}}{VoT} \tag{3. 26}$$

The variable charging station cost factor for the BEV and DBEV (Sathaye, 2014),

$$Z_{I,C} = C_c \cdot C \tag{3. 27}$$

And C_c is the variable cost per port, what is the sum of the station variable port cost, C_{C0j}, and the station maintenance variable cost, C_{C1j},

$$C_c = \frac{C_{C0j} + C_{C1j}}{VoT} \tag{3. 28}$$

Where C is the number of charging station ports (Sathaye, 2014),

$$C = \frac{M_c \cdot b_3}{\left(b_3 + \frac{d_c}{\bar{v}}\right)} \; such\ that\ C \in \mathbb{N} \tag{3. 29}$$

The cost of installed recharging posts does not count the expenses required to plan the deployment and to acquire planning permission. Nor is rental cost for parking spaces included. This decision is mainly driven by the largely varying cost per space to be seen across regions and cities. Furthermore, parking space is less of a concern for fast charging stations as opposed to level II on-street chargers (Schroeder, et al., 2012). Therefore, the cost of the recharging station for a site will be (Sathaye, 2014),

$$Z_I = C_Y \cdot Y + C_c \cdot C \tag{3. 30}$$

3.5. FLEET SIZE OR TOTAL NUMBER OF VEHICLES, M

3.5.1. Fleet size for SV

For SV in the dispatching, hailing, stand, dispatching-hailing and stand-hailing modes, taxis will have two kinds of state: in-service and empty. In the stand market, the empty state will be the one whilst taxis are back to the stand. For the present model, it is assumed the arrival of the demand and supply is constant, continuous and deterministic. Hence, using Little's Formula, the number of taxis the in-service state is M_Q and the ones empty is M_E,

$$\mu = \frac{M_Q}{T_Q} = \frac{M_E}{T_E} = \lambda_d \cdot A \tag{3. 31}$$

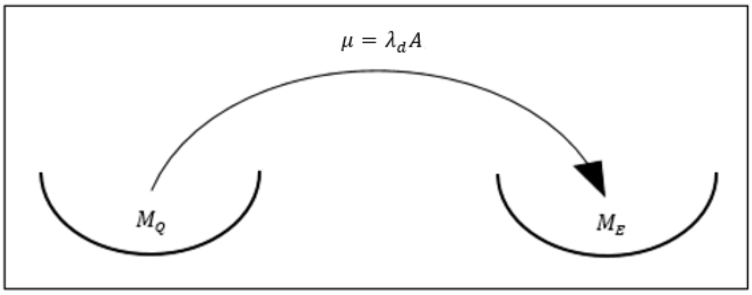


Figure 7. Little's distribution for the SV

When dispatching, it is considered it is in the in-service state, however, after servicing a trip it is considered empty state. The same happens with the stand and stand-hailing market.

Where the empty total time, T_E,

$$T_E = \frac{d_I}{v_I} \tag{3. 32}$$

This empty total time have included the dispatching time for the dispatching and dispatching-hailing markets.

The **fleet size** or the total number of taxis is,

$$M = M_Q + M_E \tag{3. 33}$$

For the total taxi cycle time, due to not right after all the services taxis will have to recharge, it will be divided by the number of services that the taxi drives without charging. So,

$$T = \frac{d_Q}{v_Q} + \frac{d_I}{v_I} \tag{3. 34}$$

3.5.1. Fleet size for BEV and DBEV

For BEV and DBEV in the dispatching, hailing, stand, dispatching-hailing and stand-hailing modes, taxis will have three kinds of state: in-service, empty and charging. The same assumptions and features mentioned above are considered. Hence, using Little's Formula and considering a number of taxis in the charging state M_C,

$$\mu = \frac{M_Q}{T_Q} = \frac{M_E}{T_E} = \frac{M_C}{T_C} = \lambda_d A \tag{3. 35}$$

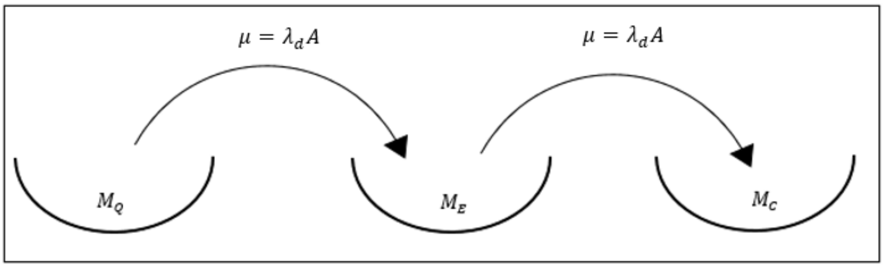


Figure 8. Little's distribution for BEV and DBEV

Where, T_C is the total charging time per cycle, what considers the ride to the charging or swapping station and right after the charging state (recharging by plug-in or switching the battery), so, the expected time spent driving to and at a charging station per cycle is,

$$T_C = \frac{2 \cdot d_C}{N \cdot v_C} + b_3 \tag{3. 36}$$

The number of taxis in the charging state (R) is equal to the number of taxis in the charging state is also equal to the number of charging taxis (M_C) divided by the expected time spent driving to and at a charging station per cycle (T_C) (Sathaye, 2014).

$$R = \frac{\lambda_d A}{N} = \frac{M_C}{T_C} \tag{3. 37}$$

The **fleet size** or the total number of taxis is,

$$M = M_Q + M_E + M_C \tag{3. 38}$$

3.6. SV, BEV AND DBEV TAXI FORMULATION. APPENDIX REFERENCE

The formulation of the user, the taxi, the infrastructure and the external cost for the SV, BEV and DBEV, is detailed in the **appendix** at the end of the present work. It is structured by two main sections:

1) SV and
2) 2) BEV and DBEV

Furthermore, each one of these sections will be composed by 5 subsections: dispatching, hailing, stand, dispatching-hailing and stand-hailing markets. Finally, each one of these markets will be detailed with: the distances and velocities, the one trip cost for one user (unitary cost), the all trip cost for all the users, the one trip cost for one taxi, the all trips cost for one taxi, the all trips cost for all taxis, the infrastructure and the external costs.

As for the Driverless with Battery Electric Vehicles, DBEV, they are marked not by a different formulation but for reduction in the hourly cost, just as an increasing of the velocity of the taxis with a passenger, v_Q, the dispatching, v_D, the stand, v_S, and the charging ones, v_C,as well as, the reduction of the hourly cost, C_h, reduces the cost of some different agencies on the different markets. With the reduction of the hourly cost, the taxi cost is direct reduced and with the increasing of the velocity the user costs is also reduced.

CHAPTER 4

MODEL ANALISYS: CASE STUDY IN BARCELONA

4.1. INTRODUCTION

The main goal of this chapter is to present the results of the integrated taxi modelling set up. This part discusses the performance of the different sorts of taxi vehicle in the all markets explained at Chapter 3.

The first line of this section aims to compare the conventional taxi vehicles (SV) with the battery electric ones (BEV), as well as the second line compares these last ones with the driverless electric taxi vehicles (DBEV). Each one of these parts is accomplished studying separately the different markets, showing the variation of the unitary system cost, z, with the taxi supply per hour and area of service, λ_d, outcomes and its sensibility with regard to the trip demand per hour and area of service, λ_u. Once plots are displayed and the optimal unitary cost, in this case the minimum, outputs are set out distinguishing the different cost values for the user, the taxi, the infrastructure and the externalities, as well as, their associated percentage weight an analysis is carried out. Afterwards, by means of the Little's Formula, the total fleet size is found.

It is important to point out the number of decimals taken in the next tables, particularly 4, in terms of the monetary units as euros [€]. It is because figures below represent unitary units and in case it is needed to know total costs this way would be useful

Reference values for the taxi supply and the trip demand per hour and area of service and **Barcelona** city parameters are used for generating comparative analyses. Values are obtained from different authors as Salanova (2015), Sathaye (2014), Kittelson (2013),

Table 5. Input values for the Barcelona case

Input variable	Units	Reference value (Barcelona)
λ_d	taxis/h·A	0-100
λ_u	trips/h·A	25; 50; 75
A	km²	100
r	-	1.7

$\bar{c}$	€/trip	9.84
VoT	€/h	20
C_{km}	€/km	0.118
C_h	€/h	20.53
v_D	km/h	25
v_Q	km/h	25
v_I	km/h	10
v_C	km/h	25
Y	Sites	(Swapping stations): 10
Y	sites	(Recharging stations): 30
b_3	h	0.42
C_s	€/site	0.25
C_{Y0j}		7.6
C_{Y1j}		3.8
C_{C0j}		1.5
C_{C1j}		0.76

4.2. SV AND BEV ANALYSIS

4.2.1. Introduction

The main purpose in this section it is to explain the benefits of the BEV regarding to the SV by means of some figures showing the advantages or disadvantages of BEV with regarding to SV. First of all, BEV need and infrastructure to recharge by plug-in or switching the battery. Secondly, total distances per trip served, d, are slightly higher for BEV than SV since they have been driven to the recharging station. Nevertheless, BEV presents lower fuel consumption, E_d and the emissions of various pollutants, F_c, what means a reduction of the pollution of the externality cost.

4.2.2. Dispatching market for SV and BEV

In the next figure (9) the unitary cost for the dispatching market is plot for a trip demand per hour and area of service of 25, 50 and 75 with a constant area of service. It can be seen how the increasing of the trip demand per hour and area of service reduces also the unitary cost and at the same time increases the minimum taxi supply associated to obtain this minimum unitary cost. Approximately when the taxi demand per hour and area of service, λ_u, is 25 and the taxi supply per hour and area of service, λ_d, is 10 there is an asymptote due to the waiting time for passenger equation at this value is indeterminate, the same happens for $\lambda_u = 50$ with an asymptote at 20 and for $\lambda_u = 75$ with an asymptote at 30. Besides, it will not be possible to have a market will a lower value of λ_d than the asymptote. Right after the minimum supply, the curve $\lambda_u = 25$ grows with the steepest slope whereas the cost curve with $\lambda_u = 75$ grows with the least, this is mainly because the taxi cost equation. On the other hand, for the dispatching market, the system unitary cost decreases when λ_u grows.

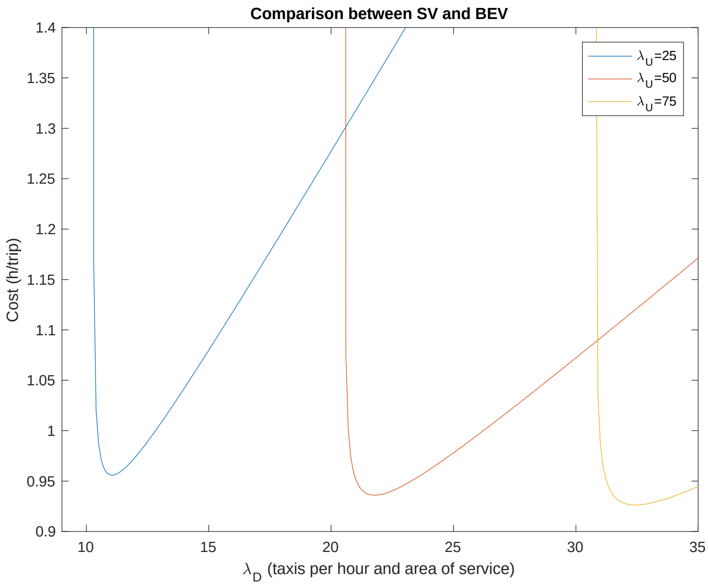


Figure 9. Comparison between SV and BEV in the dispatching market

The minimum taxi supply obtained either for BEV that work with a switching or recharging station or for SV that work with conventional fuels are equal in the dispatching market for each trip demand as we can check in the following table (6). This is because makes the difference between the SV and the BEV types is the infrastructure cost but as we can see in the table, these values are not relevant and represent at about 0.0023 h or 9 seconds.

The taxi supply needed to obtain the minimum unitary cost for the dispatching market increases while the trip demand decreases. When the trip demand increases the user unitary cost also increases percentage weight and the taxi cost loses this percentage. In this case, it is possible to see how z_u it goes from 97% with a $\lambda_u = 25$ to the 99% while z_T goes from 3% to 1%.

The BEV vehicles for the dispatching market has a higher minimum unitary cost than the SV vehicles, since an infrastructure is needed. The user cost remains equal both for BEV and SV, whereas the taxi cost is higher for the BEV than for the SV due to the distance trip will be higher since vehicles will recharge after N trips. The infrastructure cost for BEV with switching stations will be higher due to the infrastructure values but the time spent switching the battery, as we have said, will be much lower than recharging.

Comparing both SV and BEV types, there is no barely difference between the percentage weight between the user and the taxi unitary costs.

Table 6. Cost comparison between SV and BEV in the dispatching market

SV						
λ_u [trips/h·A]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
25	11.2	0.9567	0.93255	0.0241	0.0000	0.0000
			97%	3%	0%	0%
50	21.9	0.9362	0.92228	0.0139	0.0000	0.0000
			99%	1%	0%	0%
75	32.5	0.9263	0.91722	0.0091	0.0000	0.0000
			99%	1%	0%	0%

BEV with switching stations						
λ_u [trips/h·A]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
25	11.2	0.9594	0.93255	0.0252	0.0024	0.0000
			97%	3%	0%	0%
50	21.9	0.9382	0.92228	0.0149	0.0012	0.0000
			98%	2%	0%	0%
75	32.5	0.9281	0.91722	0.0101	0.0008	0.0000
			99%	1%	0%	0%
BEV with recharging stations						
λ_u [trips/h·A]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
25	11.2	0.9578	0.93255	0.0247	0.0008	0.0000
			97%	3%	0%	0%
50	21.9	0.9374	0.92228	0.0145	0.0004	0.0000
			98%	2%	0%	0%
75	32.5	0.9276	21.8735	-0.1354	0.0003	0.0000
			99%	1%	0%	0%

The following table shows the taxi fleet obtained in the dispatching market for the different taxi supplies per hour and area of service (7). These values will be the ones related with the minimum unitary cost, so, in case the minimum unitary cost of a system market is the value of design these will be the results of the fleet size. The taxi fleet obtained is slightly higher for the BEV than the SV for the same taxi supply per hour and area of service because the number of taxis in the charging state will be added while the number in service or the empty state will remain from SV to BEV. Particularly, for $\lambda_u = 25$ the number of taxis in the charging state for the BEVS system will be 6 and for the BEVR system will be 17. Hence, the BEVS system requires a higher infrastructure cost but needs less taxis than the BEVR one. Therefore, the higher fleet size obtained for the same taxi supply per hour and area of service will be for the BEV with recharging stations.

For the dispatching market, the number of vehicles to obtain the minimum cost increases when the trip demand also increases, and straightaway all the taxis in the different states.

In this case, the number of taxis in service represents the 89% for the SV type and all the trip demands, the 88% for the BEVS one and the 86% for the BEVR one. On the other hand, the percentage weight of the empty vehicles represents in all cases around the 11%. Finally, the percentage for vehicles in the charging state will be the 1% for the BEVS market and the 3%% for the BEVR one.

Table 7. Fleet size comparison between SV and BEV in the dispatching market

SV					
λ_u [trips/h·A]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]
25	11.2	427	53	0	
		89%	11%	0%	480
50	21.9	835	104	0	
		89%	11%	0%	939

75	32.5	1240	155	0	
		89%	11%	0%	1395
BEV with switching stations					
λ_u [trips/h·A]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]
25	11.2	427	53	6	
		88%	11%	1%	486
50	21.9	835	104	11	
		88%	11%	1%	950
75	32.5	1240	155	16	
		88%	11%	1%	1411
BEV with recharging stations					
λ_u [trips/h·A]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]
		427	53	17	
25	11.2	86%	11%	3%	497
		835	104	34	
50	21.9	86%	11%	3%	973
		1240	155	50	
75	32.5	86%	11%	3%	1445

For the ease of the understanding, the next bar chart (10) represents the table above,

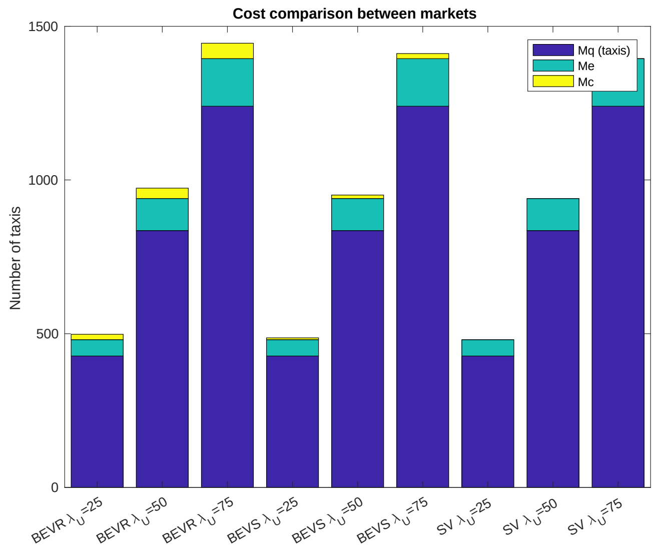


Figure 10. Bar chart for the fleet size in the dispatching market

4.2.3. Hailing market for SV and BEV

As observed in figure () the unitary cost for the dispatching market is plot for a trip demand per hour and area of service of 25, 50 and 75 with a constant area of service. It can be seen how the increasing of the trip demand per hour and area of service reduces also the unitary cost and at the same time increases the minimum taxi supply associated to obtain this minimum unitary cost. There is an asymptote at around $\lambda_d = 9$ when $\lambda_u = 25$, at $\lambda_d = 17.5$ when $\lambda_u = 50$ and at $\lambda_d = 25$ when $\lambda_u = 75$. It happens when the taxi hourly supply per hour and area of service equals the trip demand per hour and hour of service multiplied for the trip cycle time. Besides, it will not be possible to have a market will a lower value of λ_d than the asymptote. Right after the minimum supply, the curve $\lambda_u = 25$ grows with the steepest slope whereas the cost curve with $\lambda_u = 75$ grows with the least, due to the taxi cost equation. On the other hand, for the hailing market, the system unitary cost increases when λ_u grows.

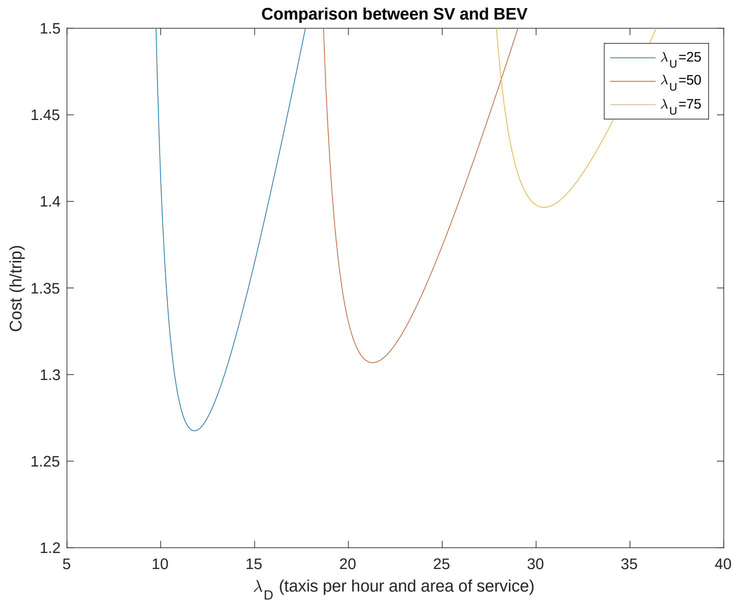


Figure 11. Comparison between SV and BEV in the hailing market

The minimum taxi supplies per hour and area of service obtained for BEV are higher than the SV types in the hailing market. Besides, the minimum taxi supplies obtained for BEV that work with a switching is slightly higher than the ones in the recharging station as we can check in the following table (8). The same happens with the unitary cost. For a $\lambda_u = 25$, the minimum unitary cost for a BEVS is 1.1231 h while for BEVR is 1.1216 h, what represents a difference of 0.0015 h or 5.4 s, however, for $\lambda_u = 75$ this difference is reduced 3.6 s. The minimum unitary cost for the SV types are higher than the BEV ones, however, the minimum taxi supply associated is lower since an infrastructure is needed for the BEV types and this function displaces de total unitary cost to the right side.

Table 8. Cost comparison between SV and BEV in the hailing market

SV						
λ_u [trips/h·A]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
			1.0302	0.0554	0.0000	0.1822
25	11.9	1.2675	81%	4%	0%	14%
			0.9879	0.0033	0.0000	0.3158
50	21.4	1.3069	78%	0%	0%	25%
			0.9724	-0.0199	0.0000	0.4442
75	30.5	1.3966	77%	2%	0%	35%

Switching stations						
λ_u [trips/h·A]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
			1.0385	0.0737	0.0024	0.0369
25	12.3	1.1231	90%	6%	0%	4%
			0.9921	0.0238	0.0012	0.0669
50	22.3	1.0536	92%	2%	0%	6%
			0.9746	0.0013	0.0008	0.0957
75	31.9	1.0392	91%	0%	0%	9%
Recharging stations						
λ_u [trips/h·A]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
			1.0205	0.0776	0.0008	0.0372
25	12.4	1.1216	90%	7%	0%	3%
			0.9692	0.0299	0.0004	0.0678
50	22.6	1.0526	91%	3%	0%	6%
			0.9461	0.0096	0.0003	0.0975
75	32.5	1.0382	91%	1%	0%	9%

The following table shows the taxi fleet obtained in the hailing market for the different taxi supplies per hour and area of service (9). These values will be the ones related with the minimum unitary cost, so, in case the minimum unitary cost of a system market is the value of design these will be the results of the fleet size. The taxi fleet obtained is slightly higher for the BEV than the SV for the same taxi supply per hour and area of service because a number of taxis in the charging state will be added, as well as the number of taxis in service or the empty state will also increase. On the other hand, the number of taxis in the charging state will be higher for the market that needs a recharging station than the one that works with a switching battery station. Therefore, the higher fleet size obtained for the same taxi supply per hour and area of service will be for the BEV with recharging stations while the unitary cost will be lowest.

The number of vehicles to obtain the minimum cost increases when the trip demand also increases, and straightaway all the taxis in the different states.

In this case, the percentage weight of the SV taxis in service increases from the 73% when $\lambda_u = 25$ to the 85%, when $\lambda_u = 75$, whereas the percentage of the empty taxis decreases from the 27% when $\lambda_u = 25$ to the 15% when $\lambda_u = 75$. These changes also happen for the BEVS and BEVR but with different values as it can be seen in the table, however, the percentage weight in the charging state remain from $\lambda_u = 25$ to $\lambda_u = 75$. The BEVR has the highest fleet size in his minimum unitary cost.

The proportion of taxis in service is higher when vehicles are SV than when they are BEV because there is no taxis in the charging state. Besides, this proportion is also higher for BEVS than BEVR, because taxis switch the battery faster (0.05 h) than taxis recharge (0.42 h). The proportion of taxis in the empty state between BEVS and BEVR will be equal.

Table 9. Fleet size comparison between SV and BEV in the hailing market

SV					
λ_u [trips/h·A]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]
25	10.8	413	154	0	567
		73%	27%	0%	
50	14.7	742	174	0	916
		81%	19%	0%	
75	20.4	1058	183	0	1241
		85%	15%	0%	
BEV with switching stations					
λ_u [trips/h·A]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]
25	11	427	179	6	612
		70%	29%	1%	
50	14.8	773	221	10	1004
		77%	22%	1%	
75	20.6	1106	251	15	1372
		81%	18%	1%	
BEV with recharging stations					
λ_u [trips/h·A]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]
25	11	430	185	17	632
		68%	29%	3%	
50	14.8	784	238	32	1054
		74%	23%	3%	
75	20.6	1127	281	45	1453
		78%	19%	3%	

For the ease of the understanding, the next bar chart (12) represents the table above,

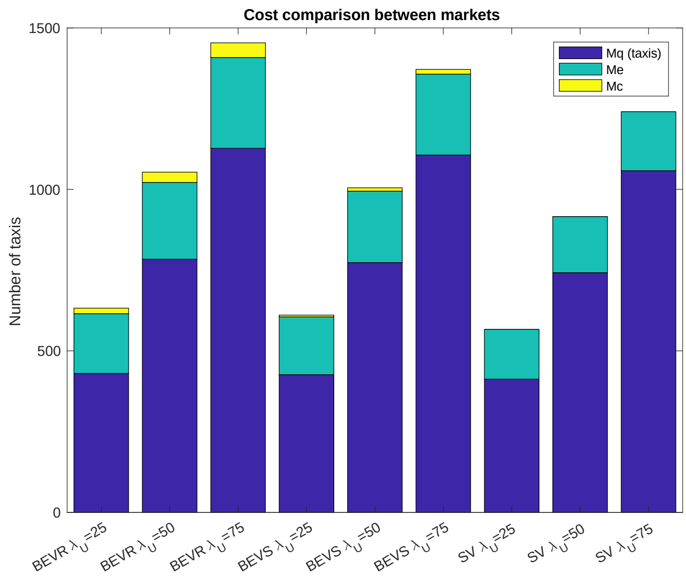


Figure 12. Bar chart for the fleet size in the hailing market

4.2.4. Stand market for SV and BEV

In the next figure (13) the unitary cost for the dispatching market is plot for a trip demand per hour and area of service of 25, 50 and 75 with a constant area of service. It can be seen how the increasing of the trip demand per hour and area of service reduces also the unitary cost and at the same time increases the minimum taxi supply associated to obtain this minimum unitary cost. There is an asymptote at $\lambda_d = 9$ when $\lambda_u = 25$, at $\lambda_d = 17.5$ when $\lambda_u = 50$ and at $\lambda_d = 27.5$ when $\lambda_u = 75$. It happens when the taxi hourly supply per hour and area of service equals the trip demand per hour and area of service multiplied for the time. This makes the access time denominator zero and therefore an indeterminate in the user cost equation. Besides, it will not be possible to have a market will a lower value of λ_d than the asymptote. Right after the minimum supply, the curve $\lambda_u = 25$ grows with the steepest slope whereas the cost curve with $\lambda_u = 75$ grows with the least, this is mainly because the taxi cost equation. On the other hand, for the stand market, the system unitary cost decreases when λ_u grows.

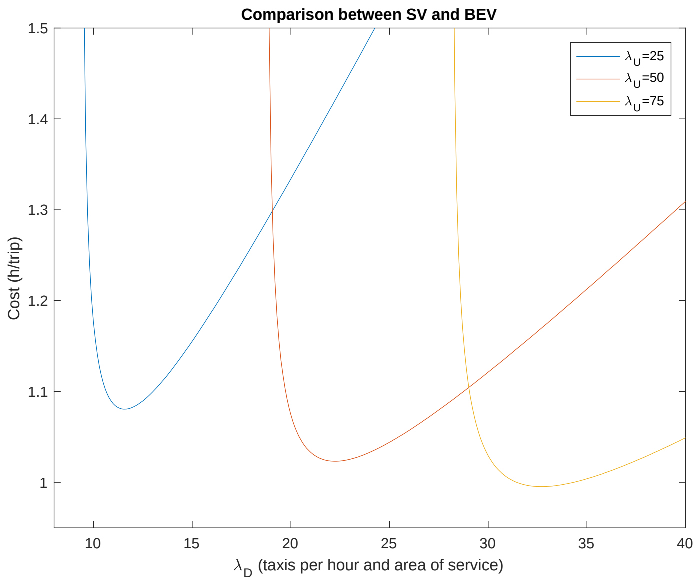


Figure 13. Comparison between SV and BEV in the stand market

Minimum taxi supplies obtained for BEV are equal than the SV types in the stand market, as well as the minimum taxi supplies obtained either for BEV that works with a switching or recharging station are equal as we can check in the following table (10). The minimum unitary cost for the SV types are equal slightly lower than the BEV ones, however, the minimum taxi supply associated is equal than the BEV since an infrastructure is needed for the BEV types.

The taxi supply needed to obtain the minimum unitary cost for the stand market increases while the trip demand decreases. When the trip demand increases, the user unitary cost for SV and BEV increases its percentage weight from 96% when $\lambda_u = 25$ to 99% when $\lambda_u = 75$ and the taxi unitary cost decreases from 4% when $\lambda_u = 25$ to 1% when $\lambda_u = 75$. The most relevant cost at the minimum point is the user.

The user cost remains equal both for BEV and SV, whereas the taxi cost is higher for the BEV than for the SV due to the distance trip will be higher since vehicles will recharge after N trips. The infrastructure cost for BEV with switching stations will be higher than the recharging ones but the time spent switching the battery, as we have said, will be much lower than recharging. Comparing both SV and BEV types in the hailing market, there is no difference between the percentage weight between the user and the taxi unitary costs.

The minimum unitary cost obtained for BEV are different from the SV types in the stand market. Nevertheless, the minimum taxi supplies obtained for BEV that work with a switching is sligightly higher than the ones in the recharging station as we can check in the following table (). For a $\lambda_u = 25$, the minimum unitary cost for a BEVS is 1.0839 h while for BEVR is 1.0823 h, what represents a difference of 0.0016 h or 5.76 s, however, for $\lambda_u = 75$ this difference is reduced 2.16 s. The minimum unitary cost for the SV types are higher than the BEV ones, however, the minimum taxi supply associated is lower since an infrastructure is needed for the BEV types and this function displaces de total unitary cost to the right side.

Table 10. Cost comparison between SV and BEV in the stand market

SV						
λ_u [trips/h·A]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
			1.03635	0.0421	0.0000	0.0000
25	11.7	1.0806	96%	4%	0%	0%
			1.00194	0.0195	0.0000	0.0000
50	22.3	1.0232	98%	2%	0%	0%
			0.98317	0.0106	0.0000	0.0000
75	32.8	0.9953	99%	1%	0%	0%
Switching stations						
λ_u [trips/h·A]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
			1.03635	0.0431	0.0031	0.0000
25	11.7	1.0839	96%	4%	0%	0%
			1.00194	0.0205	0.0022	0.0000
50	22.3	1.0254	98%	2%	0%	0%
			0.98317	0.0116	0.0019	0.0000
75	32.8	0.9972	99%	1%	0%	0%
Recharging stations						
λ_u [trips/h·A]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
			1.03635	0.0427	0.0023	0.0000
25	11.7	1.0823	96%	4%	0%	0%
			1.00194	0.0201	0.0018	0.0000
50	22.3	1.0246	98%	2%	0%	0%
			0.98317	0.0112	0.0016	0.0000
75	32.8	0.9966	99%	1%	0%	0%

The following table shows the taxi fleet obtained in the hailing market for the different taxi supplies per hour and area of service (11) where it is possible to see how the number of vehicles to obtain the minimum cost increases when the trip demand also increases, and straightaway all the taxis in the different states. The taxi fleet obtained is slightly higher for the BEV than the SV for the same taxi supply per hour and area of service because the number of taxis in the charging state will be added while the number in service or the empty state will remain from SV to BEV. On the other hand, the number of taxis in the charging state will be higher for the market that needs a recharging station than the one that works with a switching battery station. Therefore, the higher fleet size obtained for the same taxi supply per hour and area of service will be for the BEVR while the unitary cost will be lower than the BEVS ones. The BEVR has the highest fleet size in his minimum unitary cost.

For the stand market, the number of taxis in service represents the 89% for the SV type and all the trip demands, the 88% for the BEVS one and the 86% for the BEVR one. On the other hand, the percentage weight for the empty vehicles represents the 11% for the SV and BEVS and the 10% for the BEVR. Finally, the percentage for vehicles in the charging state will be 1% for the BEVR market and less relevant for the SV and BEVS types.

Table 11. Fleet size comparison between SV and BEV in the stand market

SV					
λ_u [trips/h·A]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]
25	7.1	426	51	0	477
		89%	11%	0%	
50	9.6	812	97	0	909
		89%	11%	0%	
75	13.2	1194	142	0	1336
		89%	11%	0%	
BEV with switching stations					
λ_u [trips/h·A]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]
25	7.1	426	51	6	483
		88%	11%	1%	
50	9.6	812	97	11	920
		88%	11%	1%	
75	13.2	1194	142	16	1352
		88%	11%	1%	
BEV with recharging stations					
λ_u [trips/h·A]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]
25	7.1	426	51	17	494
		86%	10%	3%	
50	9.6	812	97	33	942
		86%	10%	4%	
75	13.2	1194	142	48	1384
		86%	10%	3%	

For the ease understanding, the next bar chart (14) represents the table above,

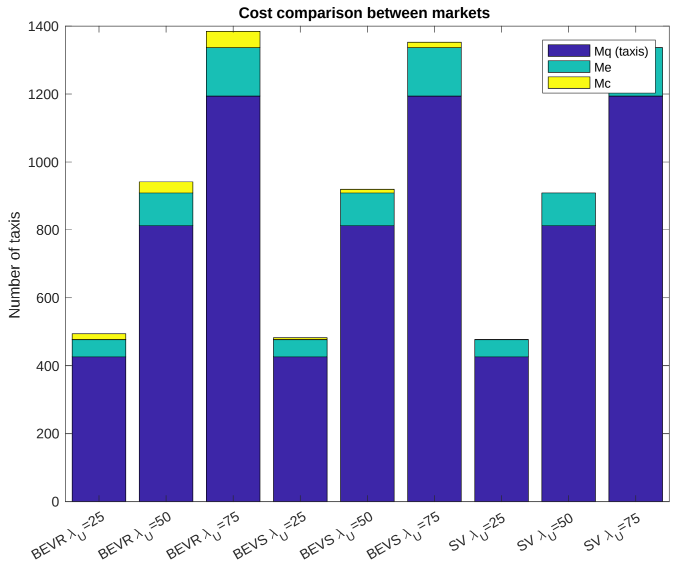


Figure 14. Bar chart for the fleet size in the stand market

4.2.5. Dispatching-hailing market for SV and BEV

As observed in figure (15) the unitary cost for the dispatching market is plot for a trip demand per hour and area of service of 25, 50 and 75 with a constant area of service. It can be seen how the increasing of the trip demand per hour and area of service reduces also the unitary cost and at the same time increases the minimum taxi supply associated to obtain this minimum unitary cost. There is also an asymptote at $\lambda_d = 7.5$ when $\lambda_u = 25$, at $\lambda_d = 17$ when $\lambda_u = 50$ and at $\lambda_d = 27.5$ when $\lambda_u = 75$. It happens when the taxi hourly supply per hour and area of service equals the trip demand per hour and hour of service multiplied for the time. This makes the combination of the waiting times of the user cost equation denominator zero and as a consequence indeterminate in the user cost equation. Besides, it will not be possible to have a market will a lower value of λ_d than the asymptote. Right after the minimum supply, the curve $\lambda_u = 25$ grows with the steepest slope whereas the cost curve with $\lambda_u = 75$ grows with the least, this is mainly because the taxi cost equation.

The dispatching-hailing market analysis conducted in this section represents a combination of these two markets joint by the parameter γ (Sathaye). As it is possible to see in the next figure the system unitary cost increases when λ_u grows what this is a feature of the hailing market over the dispatching. This is because this study has considered a higher parameter involvement for the hailing than the dispatching.

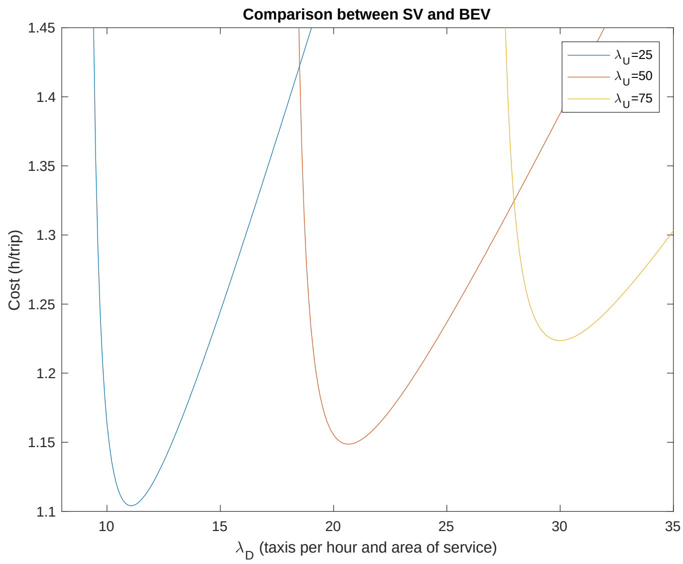


Figure 15. Comparison between SV and BEV in the dispatching-hailing market

The minimum taxi supply obtained for BEV is higher than the SV types in the dispatching-hailing market. Nevertheless, the minimum taxi supply obtained either for BEV that work with a switching or recharging station are equal as we can check in the following table (12). The minimum total cost for the SV types are higher than the BEV ones, however, the minimum taxi supply associated is lower since an infrastructure is needed for the BEV types.

The minimum taxi unitary costs obtained for BEV are different from the SV types in the dispatching-hailing market, as well as the BEVR has a lower unitary cost than the BEVS, being the lowest. For a $\lambda_u = 25$, the minimum unitary cost for a BEVS is 1.0839 h while for BEVR is 1.0823 h, what represents a difference of 0.0016 h or 5.76 s, however, for $\lambda_u = 75$ this difference is reduced 2.16 s. The minimum unitary cost for the SV types are higher than the BEV ones, however, the minimum taxi supply associated is lower since an infrastructure is needed for the BEV types and this function displaces de total unitary cost to the right side

Table 12. Cost comparison between SV and BEV in the dispatching-hailing market

SV						
λ_u [trips/h·A]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
			0.97712	-0.1320	0.0000	0.8053
25	11.2	1.1041	51%	7%	0%	42%
			0.96298	-0.1432	0.0000	0.8036
50	20.8	1.1487	50%	8%	0%	42%
			0.95351	-0.1561	0.0000	0.8016
75	30.1	1.2236	50%	8%	0%	42%
Switching stations						
λ_u [trips/h·A]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
			0.97328	-0.6024	0.0024	0.7106
25	11.7	1.0839	43%	26%	0%	31%
			0.96028	-0.6141	0.0018	0.6775
50	22.3	1.0254	43%	27%	0%	30%

			0.94955	-0.6249	0.0013	0.6713
75	32.8	0.9972	42%	28%	0%	30%
Recharging stations						
λ_u [trips/h·A]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
			0.97328	-0.6043	0.0002	0.7131
25	11.7	1.0823	42%	26%	0%	31%
			0.96028	-0.6155	0.0002	0.6796
50	22.3	1.0246	43%	27%	0%	30%
			0.94955	-0.6259	0.0001	0.6728
75	32.8	0.9966	42%	28%	0%	30%

The following table shows the taxi fleet obtained in the hailing market for the different taxi supplies per hour and area of service (13). These values will be the ones related with the minimum unitary cost, so, in case the minimum unitary cost of a system market is the value of design these will be the results of the fleet size. The taxi fleet obtained is slightly higher for the BEV than the SV for the same taxi supply per hour and area of service because the number of taxis in the charging state will be added while the number in service or the empty state will remain from SV to BEV. On the other hand, the number of taxis in the charging state will be higher for the market that needs a recharging station than the one that works with a switching battery station. Therefore, the higher fleet size obtained for the same taxi supply per hour and area of service will be for the BEV with recharging stations.

The number of vehicles to obtain the minimum cost increases when the trip demand also increases, and straightaway all the taxis in the different states.

For the dispatching-hailing market, the number of taxis in service represents the 89% for the SV type and all the trip demands, the 88% for the BEVS one and the 86% for the BEVR one. On the other hand, the percentage weight for the empty vehicles represents the 11% for the SV and BEVS and the 10% for the BEVR. Finally, the percentage for vehicles in the charging state will be 1% for the BEVR market and less relevant for the SV and BEVS types. The BEVR has the highest fleet size in his minimum unitary cost.

The proportion of taxis in service is higher when vehicles are SV than when they are BEV because there is no taxis in the charging state. Besides, this proportion is also higher for BEVS than BEVR, because taxis switch the battery faster (0.05 h) than taxis recharge (0.42 h). The proportion of taxis in the empty state between BEVS and BEVR will be equal.

Table 13. Fleet size comparison between SV and BEV in the dispatching-hailing market

SV					
λ_u [trips/h·A]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]
		426	51	0	
25	10.8	89%	11%	0%	477
		812	97	0	
50	14.7	89%	11%	0%	909
		1194	142	0	
75	20.3	89%	11%	0%	1336

BEV with switching stations					
λ_u [trips/h·A]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]
25	10.9	426	51	6	483
		88%	11%	1%	
50	14.8	812	97	11	920
		88%	11%	1%	
75	20.5	1194	142	16	1352
		88%	11%	1%	
BEV with recharging stations					
λ_u [trips/h·A]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]
25	10.9	426	51	17	494
		86%	10%	3%	
50	14.8	812	97	33	942
		86%	10%	4%	
75	20.5	1194	142	48	1384
		86%	10%	3%	

For the ease understanding, the next bar chart (16) represents the table above,

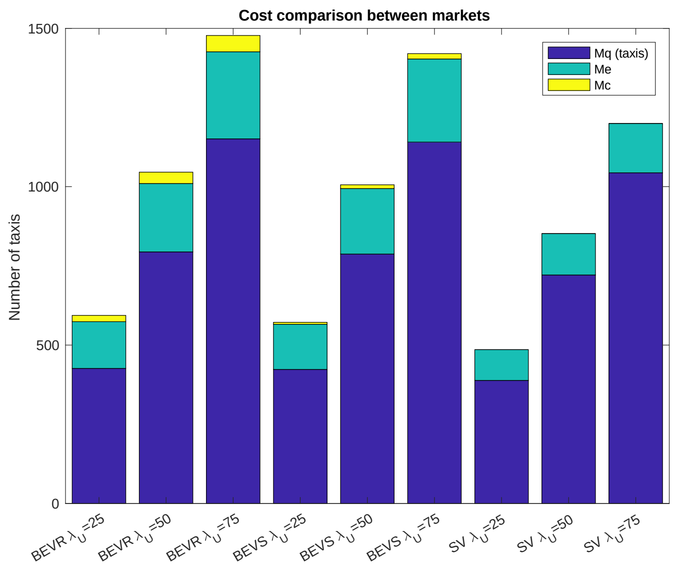


Figure 16. Bar chart for the fleet size in the dispatching-hailing market

4.2.6. Stand-hailing market for SV and BEV

In the next figure (17) the unitary cost for the dispatching market is plot for a trip demand per hour and area of service of 25, 50 and 75 with a constant area of service. It can be seen how the increasing of the trip demand per hour and area of service increases the unitary cost and at the same time increases the minimum taxi supply associated to obtain this minimum unitary cost. There is also an asymptote at $\lambda_d = 9$ when $\lambda_u = 25$, at $\lambda_d = 17.5$ when $\lambda_u = 50$ and at $\lambda_d = 27.5$ when $\lambda_u = 75$. It happens when the taxi hourly supply per hour and area of service equals the trip demand per hour and hour of service multiplied for the time. This makes the combination of waiting and access times of the user cost equation denominator zero and as a consequence indeterminate in the user cost equation. Besides, it will not be possible to have a market will a lower value of λ_d than the asymptote. Right after the minimum supply, the curve $\lambda_u = 25$ grows with the steepest slope whereas the cost curve with $\lambda_u = 75$ grows with the least, this is mainly because the taxi cost equation.

The stand-hailing market analysis conducted in this section represents a combination of these two markets joint by the parameter γ (Sathaye). As it is possible to see in the next figure the system unitary cost increases when λ_u grows what this is a feature of the hailing market over the stand. This is because the study has considered a higher parameter involvement for the hailing than the stand.

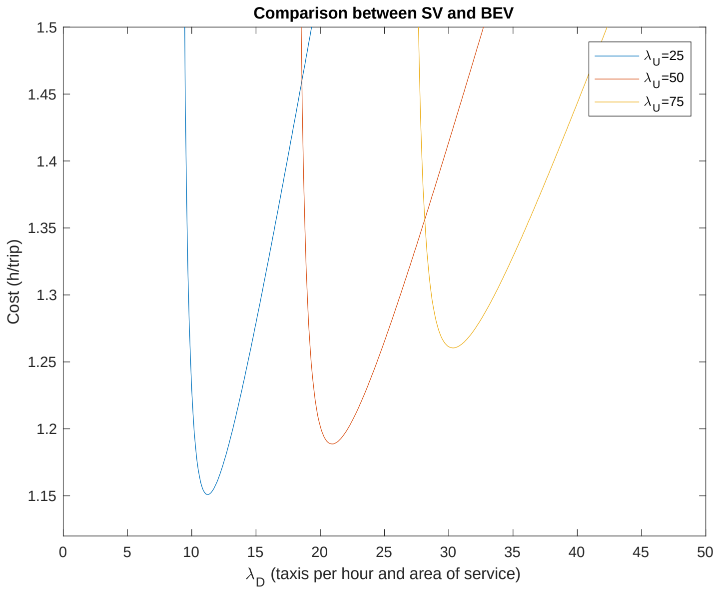


Figure 17. Comparison between SV and BEV in the stand-hailing market

The minimum taxi supply obtained for BEV is higher than the SV types in the stand-hailing market. Nevertheless, the minimum taxi supply obtained either for BEV that work with a switching or recharging station are equal as we can check in the following table (14). The minimum total cost for the SV types are higher than the BEV ones, however, the minimum taxi supply associated is lower since an infrastructure is needed for the BEV types.

The minimum taxi unitary costs obtained for BEV are different from the SV types in the dispatching-hailing market, as well as the BEVR has a lower unitary cost than the BEVS, being the lowest. For a $\lambda_u = 25$, the minimum unitary cost for a BEVS is 1.0422 h while for BEVR is 1.0405 h, what represents a difference of 0.0017 h or 6.12 s, however, for $\lambda_u = 75$ this difference is reduced 2.16 s. The minimum unitary cost for the SV types are higher than the BEV ones, however, the minimum taxi supply associated is lower since an infrastructure is needed for the BEV types and this function displaces de total unitary cost to the right side.

Table 14. Cost comparison between SV and BEV in the stand-hailing market

SV						
λ_u [trips/h·A]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
			0.98193	-0.0040	0.0010	0.1720
25	11.3	1.1510	85%	0%	0%	15%
			0.95085	-0.0720	0.0007	0.3092
50	21	1.1887	71%	5%	0%	23%

			0.93759	-0.1201	0.0005	0.4424
75	30.4	1.2604	62%	8%	0%	29%
Switching stations						
λ_u [trips/h·A]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
			0.97315	0.0311	0.0034	0.0345
25	11.5	1.0422	93%	3%	0%	4%
			0.94345	-0.0118	0.0019	0.0639
50	21.3	0.9975	92%	1%	0%	7%
			0.9287	-0.0337	0.0014	0.0927
75	30.9	0.9891	88%	3%	0%	9%
Recharging stations						
λ_u [trips/h·A]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
			0.97315	0.0316	0.0013	0.0345
25	11.5	1.0405	94%	3%	0%	3%
			0.94345	-0.0115	0.0009	0.0639
50	21.3	0.9967	93%	1%	0%	6%
			0.9287	-0.0335	0.0007	0.0927
75	30.9	0.9885	88%	3%	0%	9%

The following table shows the taxi fleet obtained in the hailing market for the different taxi supplies per hour and area of service (15). These values will be the ones related with the minimum unitary cost, so, in case the minimum unitary cost of a system market is the value of design these will be the results of the fleet size. The taxi fleet obtained is slightly higher for the BEV than the SV for the same taxi supply per hour and area of service because the number of taxis in the charging state will be added while the number in service or the empty state will remain from SV to BEV. On the other hand, the number of taxis in the charging state will be higher for the market that needs a recharging station than the one that works with a switching battery station. Therefore, the higher fleet size obtained for the same taxi supply per hour and area of service will be for the BEV with recharging stations.

The number of vehicles to obtain the minimum cost increases when the trip demand also increases, and straightaway all the taxis in the different states.

For the stand-hailing market and $\lambda_u = 25$, the number of taxis in service represents the 55% for the SV type and all the trip demands, the 54% for the BEVS one and the 53% for the BEVR one, what means they are very close. For $\lambda_u = 50$ this value is increased just 1% and another one for $\lambda_u = 75$ for most of the values. On the other hand for $\lambda_u = 25$, the percentage weight for the empty vehicles represents the 45% for the SV, 54% for BEVS and 53% for the BEVR. As it has happened for the taxis in service, for $\lambda_u = 50$ this value is increased just 1% and another one for $\lambda_u = 75$ for most of the values. Finally, the percentage for vehicles in the charging state will be 4-5%% for the BEVR market and less relevant for the SV and BEVS types. The BEVR has the highest fleet size in his minimum unitary cost.

The proportion of taxis in service is higher when vehicles are SV than when they are BEV because there is no taxis in the charging state. Besides, this proportion is also higher for BEVS than BEVR, because taxis switch the battery faster (0.05 h) than taxis recharge (0.42 h). The proportion of taxis in the empty state between BEVS and BEVR will be equal.

Table 15. Fleet size comparison between SV and BEV in the stand-hailing market

SV					
λ_u [trips/h·A]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]
		187	153	0	
25	11.3	55%	45%	0%	340
		256	200	0	
50	21	56%	44%	0%	456
		360	276	0	
75	30.4	57%	43%	0%	636
BEV with switching stations					
λ_u [trips/h·A]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]
		187	153	5	
25	11.5	54%	44%	1%	345
		257	205	7	
50	21.3	55%	44%	1%	469
		360	276	10	
75	30.9	56%	43%	2%	646
BEV with recharging stations					
λ_u [trips/h·A]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]
		187	153	16	
25	11.5	53%	43%	4%	356
		257	205	22	
50	21.3	53%	42%	5%	484
		360	276	31	
75	30.9	54%	41%	5%	667

For the ease understanding, the next bar chart (18) represents the table above,

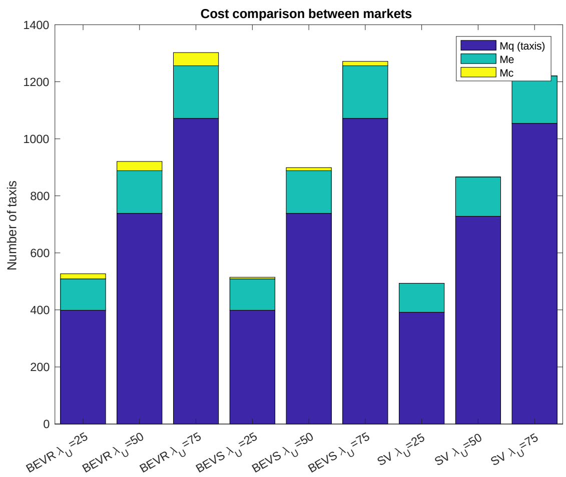


Figure 18. Bar chart for the fleet size in the stand-hailing market

4.2.7. General comparison

The analysis conducted below is a comparison between different markets studied above with the same trip demand, $\lambda_u = 50$. Once it has been seen there is no major difference between the SV, BEVS and BEVR, it will be useful for the ease of the evaluation to plot these different markets just for SV.

In the next figure (19), it can be seen the behavior of the unitary cost according to the taxi supply per hour and area of service, λ_d. It is interesting to appreciate that a lower cost does not mean a lower taxi supply per hour and area of service. For example, the dispatching market will have the lowest minimum unitary cost but the dispatching-hailing market will have the lowest taxi supply per hour and area of service. In case the taxi supply per hour and area of service is already known, the lowest unitary system cost will be provided by the stand-hailing market from $\lambda_d = 18$ to $\lambda_d = 20.5$. As of this last value, the most optimum market will be the dispatching one.

As it has been mentioned before, the lowest minimum unitary cost, will be provided by the dispatching market, whereas, the dispatching-hailing market will have the highest. On the other hand, the dispatching and hailing market will have the lowest taxi supply per hour and area of service, λ_d, and the stand market the highest.

In the foregoing sections it has been described how asymptotes demarcate a minimum possible taxi supply per hour and service. Particularly, the stand-hailing market will have the highest range, starting from $\lambda_d = 18$, followed by the hailing, the stand, the dispatching-hailing and having the highest demarcation, the dispatching market. We can conclude that for cities with a high taxi supply per hour and area of service, λ_d, this market will be the most efficient one, however, for cities that have low taxi supplies this market will not work.

In case of a city that already have the fleet size and therefore the taxi supply per hour and area of service it is possible to see which one will be the most optimal market in function of the fleet size per hour and area of service. Lower λ_d than 17 will not be possible in any market. If λ_d goes from 17 to 19.4 the most optimal market will be also the stand-hailing followed by the hailing. From 19.4 to 20.6 the most optimal market will be also the stand-hailing, followed in this section by the stand market. From 20.6 to 20.7 the stand-hailing market will continue being the market with the lowest system costs followed in this case by the dispatching market that from 27 will be the best option hereinafter, followed by the stand-hailing, the stand and the hailing market, all of them with a unitary cost value between 1.1 and 1.5 h. However, the dispatching-hailing market presents way higher values at around 1.26 h. All unitary costs values barely do not change from around $\lambda_d = 21$.

Hence, for cities with a high taxi supply the dispatching mode will be the ideal, however, for cities with a low rate of taxi supply the stand-hailing will be the best. Besides, in cities with a certain value of λ_d some markets mill not be possible.

In the foregoing sections it has been explained how the unitary cost varies over the trip demand per hour and area of service, λ_u, if this cost decreases or increases when that demand increases and vice versa. In case λ_u increases, the dispatching and the stand will decrease, however the hailing, the dispatching-hailing and the stand-hailing will increase, what means that for cities with a high λ_u the stand market will be the most optimal market for low λ_d and the dispatching one will be the best for high λ_d.

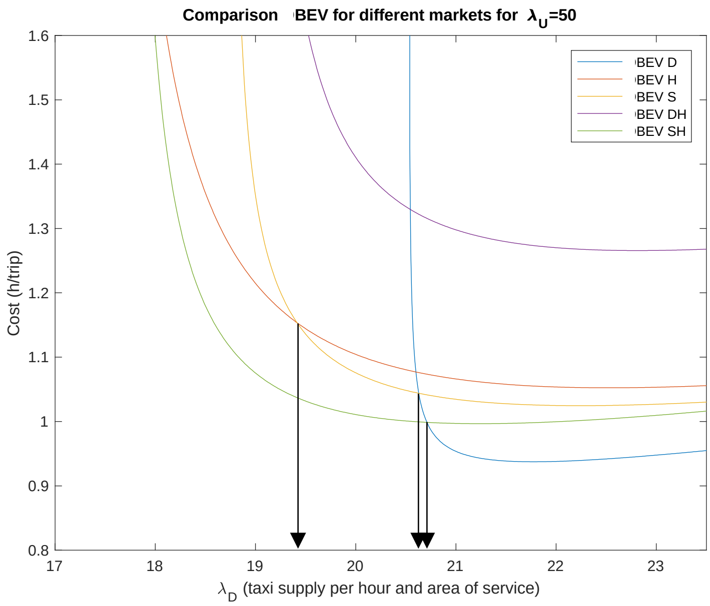


Figure 19. Cost comparison between markets for BEV

The following table (16) represents the cost for the different markets. These values will be the ones related with the minimum unitary cost. The dispatching and the stand market are the ones that have the highest percentage of the user cost in the SV and BEV, whereas the stand-hailing market has the lowest one. On the other hand, for the SV vehicles, the dispatching-hailing and stand-hailing systems have the highest taxi unitary cost weight percentage, while the hailing is the lowest.

For all the systems the infrastructure cost does not reach the 1% in percentage weight. In case of the city cost, z_c, the hailing will have the highest percentage followed by the dispatching-hailing and the stand-hailing.

Table 16. Cost comparison between markets for the SV and BEV

	SV					
	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
D	21.8	0.9360	0.9244	0.0116	0.0000	0.0000
			99%	1%	0%	0%
H	21.3	1.3069	0.9915	0.0011	0.0000	0.3143
			76%	0%	0%	24%
S	22.3	1.0232	1.0030	0.0185	0.0018	0.0000
			98%	2%	0%	0%
DH	20.7	1.3251	1.1090	-0.0887	0.0000	0.3048
			74%	6%	0%	20%
SH	20.9	1.1887	0.9528	-0.0729	0.0007	0.3081
			71%	6%	0%	23%

	BEV with switching stations					
	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
			0.9244	0.0126	0.0012	0.0000
D	21.8	0.9382	99%	1%	0%	0%
			0.9955	0.0218	0.0012	0.0666
H	22.2	1.0536	92%	2%	0%	6%
			1.0030	0.0195	0.0022	0.0000
S	22.3	1.0254	98%	2%	0%	0%
			1.0714	-0.1836	0.0013	0.0679
DH	22.6	1.1926	81%	14%	0%	5%
			0.9444	-0.0126	0.0019	0.0638
SH	21.3	0.9975	92%	2%	0%	6%
	BEV with recharging stations					
	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
			0.9244	0.0122	0.0004	0.0000
D	21.8	0.9374	99%	1%	0%	0%
			0.9711	0.0284	0.0004	0.0676
H	22.5	1.0524	91%	3%	0%	6%
			1.0030	0.0191	0.0018	0.0000
S	22.3	1.0246	98%	2%	0%	0%
			1.0983	-0.1375	0.0001	0.0685
DH	22.8	1.2657	84%	11%	0%	5%
			0.9444	-0.0123	0.0008	0.0638
SH	21.3	0.9967	92%	2%	0%	6%

The next table (17) will show the fleet size required for all the markets. These values will be the ones related with the minimum unitary cost, so, in case the minimum unitary cost of a system market is the value of design these will be the results of the fleet size. For the SV, the dispatching market will have the highest number of vehicles to cover the minimum unitary cost, while the dispatching-hailing market will have the lowest followed by the stand-hailing. In case of the BEV, the dispatching-hailing market will have the highest number of taxis and the stand the lowest.

Table 17. Fleet size comparison between markets for SV and BEV

	SV				
	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]
		831	104	0	935
D	21.8	89%	11%	0%	100%
		739	169	0	908
H	21.3	81%	19%	0%	100%
		810	96	0	906
S	22.3	89%	11%	0%	100%

		716	125	0	841
DH	20.7	85%	15%	0%	100%
		726	135	0	861
SH	20.9	84%	16%	0%	100%
	BEV with switching stations				
	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]
		831	104	11	946
D	21.8	88%	11%	1%	100%
		770	216	10	996
H	22.2	77%	22%	1%	100%
		810	96	11	917
S	22.3	88%	10%	1%	100%
		785	204	12	1001
DH	22.6	78%	20%	1%	100%
		737	148	11	896
SH	21.3	82%	17%	1%	100%
	BEV with recharging stations				
	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]
		831	104	33	968
D	21.8	86%	11%	3%	100%
		781	234	31	1046
H	22.5	75%	22%	3%	100%
		810	96	33	939
S	22.3	86%	10%	4%	100%
		791	212	36	1039
DH	22.8	76%	20%	3%	100%
		737	148	32	917
SH	21.3	80%	16%	3%	100%

For the ease understanding, the next bar chart (20) represents the table above,

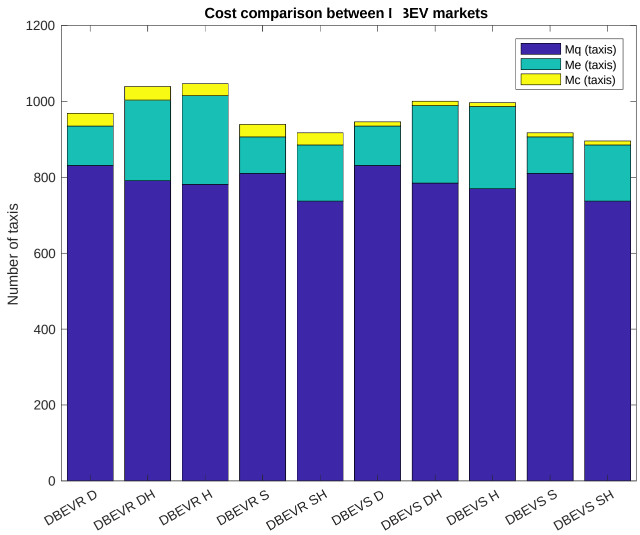


Figure 20. Bar chart for comparing fleet size between markets for BEV

The following bar chart (21) shows the total cycle time per trip served by a taxi. The dispatching market will have the highest cause the taxi must move to the passenger, pick them up and afterwards go to the passenger destination, On the other hand, the stand-hailing the lowest cause this system mixes the stand, where the passenger goes to the taxi while this is stopped. Some readers could think the stand market could have had the lowest one, but the combination with the hailing one will avoid the fact to finish their way back to the stand to wait for a passenger

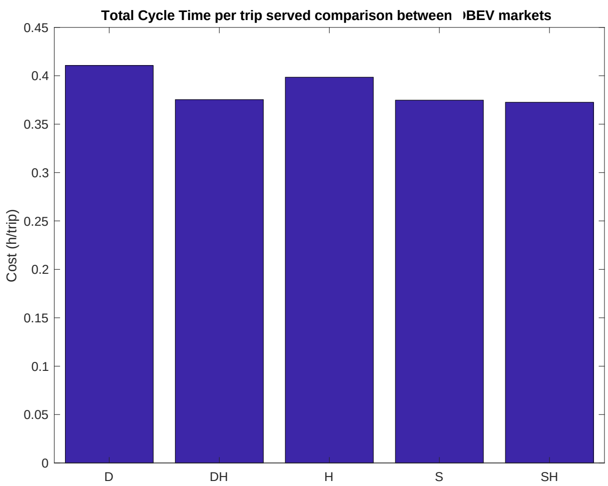


Figure 21. Total cycle time per trip served comparison between BEV markets

All distances per trip will be very similar between markets, as it is possible to see in next bar chart (22), at around 9 km, however the stand market will be the lowest one due to they will be stopped a longer time

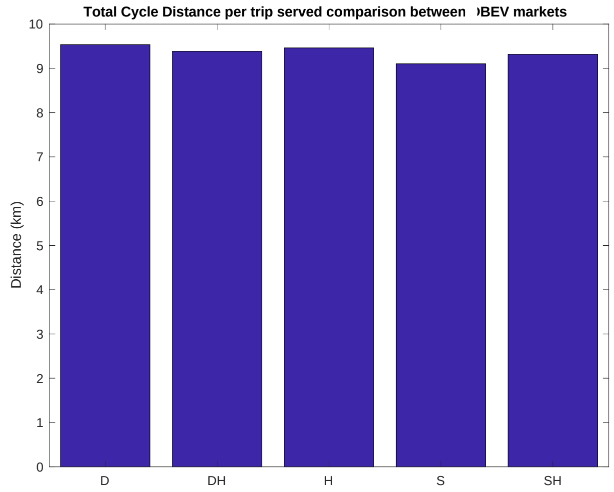


Figure 22. Total cycle distance per trip served comparison between BEV markets

The following chart (23) shows how the hailing market has the highest idle distance followed the dispatching-hailing and stand-hailing. The stand market will have the lowest.

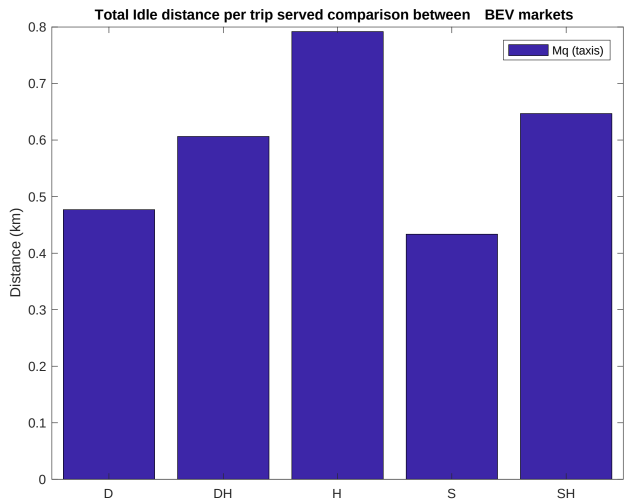


Figure 23. Total idle distance per trip served comparison between BEV markets

With regard to the user cost, it will follow the next shape as the figure (24) shows. It is interesting to see how these curves are mainly featured by the waiting time, T_W, in the dispatching and hailing markets, the access time, T_a, in the stand market and a mix of both of them for the dispatching-hailing and stand-hailing markets.

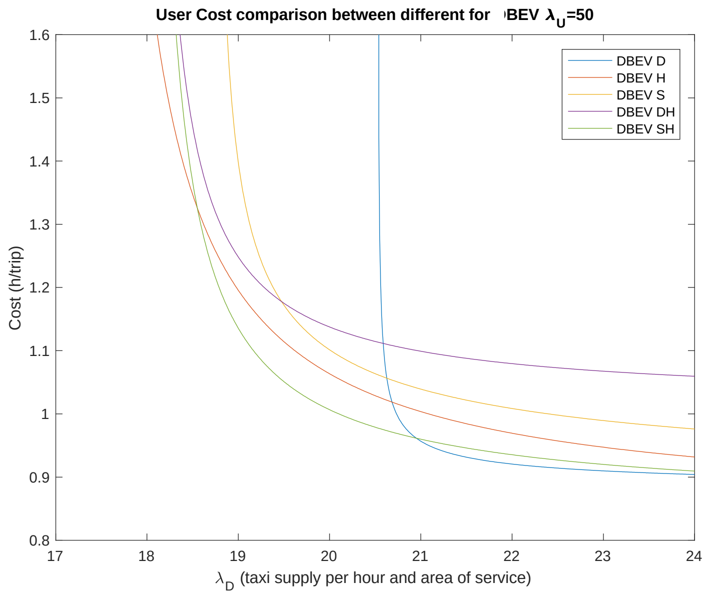


Figure 24. User cost comparison between different markets for BEV

The taxi cost will be linear as observed in figure (25). The total cycle distance will define the separation between figures, consequently the stand market equation will cute with the lowest point in the ordinate axis due to it has the lowest distance. Besides, the dispatching-hailing market will be very close with the stand-hailing.

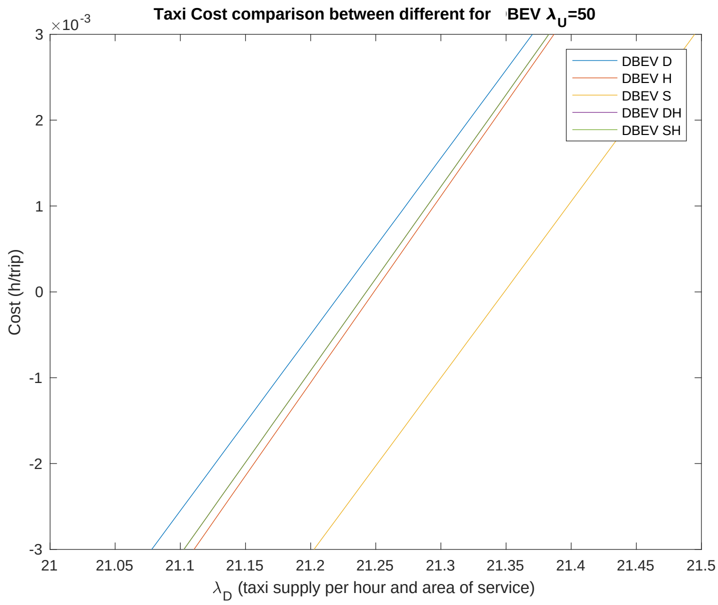


Figure 25. Taxi cost comparison between different markets for BEV

4.3. BEV AND DBEV ANALYSIS

4.3.1. Introduction

The main goal in this section it will be to present with figures the benefits of the DBEV regarding to the BEV, understanding how the increasing of the velocity of the taxis with a passenger, v_Q, the dispatching, v_D, the stand, v_S, and the charging ones, v_C,as well as, the reduction of the hourly cost, C_h, reduces the cost of some different agencies on the different markets (From now on for the ease of the reader, these velocities will be considered just as velocity, v). Afterwards, the optimum taxi supply per hour and area of service, λ_d, will be obtained from the cost functions and therefore, the optimal number of taxis will be solved through this optimal taxi supply and the Little's formula explained before.

As it will be seen in the following sections, the more velocity due to the new driverless car technologies, the less unitary cost and the less minimum taxi supply (not necessary optimal) in a particular market. However, between markets, the less unitary cost does not mean the less minimum supply.

In the hourly cost decreasing case, the less hourly cost but the more taxi supply necessary.

4.3.2. DBEV: reduction of the cruising velocity and the hourly cost

4.3.2.1. Dispatching market for DBEV

In the next figure (26) the unitary cost for the dispatching market is plot for a velocity of 25, 30 and 35 m/s with a constant city area, A, and demand for taxi trips per hour and area, λ_u. It can be seen how the reduction of the velocity reduces also the unitary cost and at the same time decreases the minimum taxi supply associated. Right after the minimum supply, all curves grow with the same slope given that a velocity change modifies the user cost and not the taxi one.

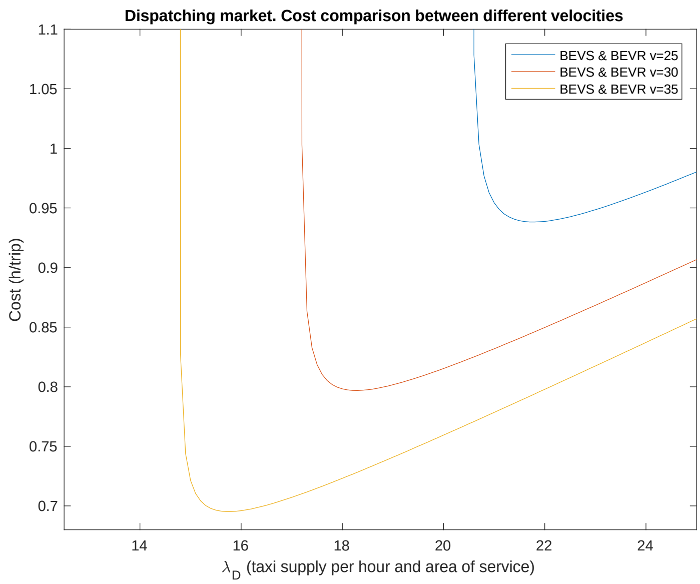


Figure 26. DBEV cost comparison between different velocities in the dispatching market

The minimum taxi supply obtained either for vehicles that works with a switching or recharging station are equal as it is observed in the next table (18) because the main difference is due because the infrastructure cost, what is not very significant in both cases, as well as the difference between them plays a minor role. When the velocity is 25 m/s the user costs represents the 98% of the total one, while when the velocity reaches 35 m/s this percentage decreases to 88%. Moreover, the taxi cost percentage weight increases from 2% to 7-12%. The infrastructure cost is higher for switching stations than for the recharging ones but the percentage weight barely reaches the 0% for both of them. There is no city cost as it is not considered as a hypothesis. Besides, we can see how for the BEVS the total cost is somewhat a 0.0008 h lower than the BEVR because of the infrastructure cost.

Table 18. DBEV cost comparison between different velocities in the dispatching market

Switching stations						
v [km/h]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
			0.92228	0.0149	0.00122	0
25	21.9	0.9382	98%	2%	0%	0%
			0.85281	-0.057	0.00121	0
30	18.4	0.7969	94%	6%	0%	0%
			0.80261	-0.1083	0.0012	0
35	15.9	0.6953	88%	12%	0%	0%
Recharging stations						
v [km/h]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
			0.9223	0.0145	0.0004	0
25	21.9	0.93742	98%	2%	0%	0%
			0.85281	-0.0574	0.0004	0
30	18.4	0.79607	94%	6%	0%	0%
			0.7859	-0.0574	0.0004	0
35	15.9	0.69446	93%	7%	0%	0%

The next plot shows the variation of the unitary cost due to the taxi supply per hour and area of service for different hourly costs, C_h=20, 17.5 and 15, with a constant hourly demand per hour and area, λ_u, city area, A, and velocity, v, of 25 m/s. As it is possible to appreciate, the less hourly cost needed, the less unitary cost but the more taxi supply per hour and area of service needed, λ_d. This is because a reduction in the hourly cost implies a reduction of the taxi cost while other agents remain and a new equilibrium is found what requires more taxi supply but not necessarily more vehicles as it will be analyzed immediately thereafter. As it has happened before, there is an asymptote at 20.6 what remains for all the equations due to there is no change in the user cost. Right after the minimum supply per hour and area of service different slopes grow given that an hourly cost change modifies the user cost slope. The difference of distance between the approximately straight curves is due because of the cut-off point of the taxi cost function with the ordinates axis what is proportional with the hourly cost, C_h.

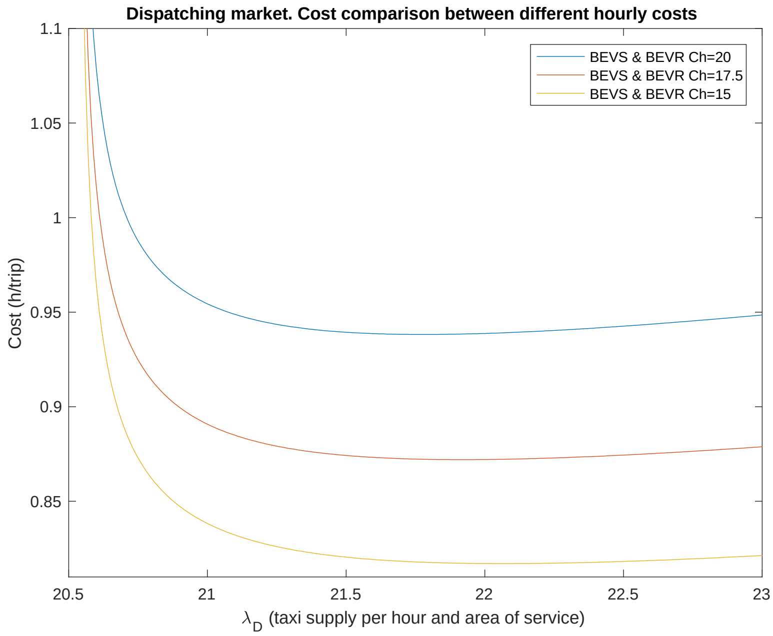


Figure 27. DBEV Cost comparison between different hourly costs in the dispatching market

The minimum taxi supply per hour and area obtained either for vehicles that works with a switching or recharging station are equal as it is observed in the following table (19). In this case, the number of taxis found is higher when the hourly cost decreases. Whether the hourly cost decreases, there is a change in the hourly cost, C_h, when the user cost percentage weight decreases while the taxi cost increases and the infrastructure cost remain. Summarizing, in the new driverless vehicles, DBEV, on the dispatching market, the user cost weight will be decreased just because of the increasing velocity.

Table 19. DBEV Cost comparison between different hourly costs in the dispatching market

Switching stations						
C_h [€/h]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
			0.9244	0.0126	0.0012	0.0000
20.0	21.8	0.9382	99%	1%	0%	0%
			0.9218	-0.0509	0.0012	0.0000
17.5	21.9	0.8720	95%	5%	0%	0%
			0.9194	-0.1035	0.0012	0.0000
15.0	22.1	0.8171	90%	10%	0%	0%
Recharging stations						
C_h [€/h]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
			0.9244	0.0122	0.0004	0.0000
20.0	21.8	0.9374	99%	1%	0%	0%
			0.9218	-0.0514	0.0004	0.0000
17.5	21.9	0.8712	95%	5%	0%	0%
			0.9218	-0.1062	0.0004	0.0000
15.0	22.1	0.8163	90%	10%	0%	0%

Once the minimum taxi supply is found it is possible to obtain the number of vehicles needed for the DBEV. As observed in the next table (20) the distribution of vehicles, distinguishing between those who are carrying a passenger, M_Q, those who are empty, M_E,and those who are in the recharging state, M_C. There is an important decreasing of the amount of taxis needed when velocity is reduced both for BEVS and BEVR, from a velocity of 25 m/s to 30 m/s, the fleet needed is reduced a 28% while from a 30 m/s to 35 m/s, the reduction is the 24% as we can see in the table (). Therefore, there is an exponential decreasing of the number of taxis when the velocity increases. On the other hand, the number of vehicles in the charging state is higher for the BEVR than for the BEVS, because of the time in the charging state for BEVS is 0.37 h lower. Likewise, a reduction in the velocity, gives a higher amount of taxis in the empty state, decreasing the ones carrying a passenger. This is due because the taxi trip will be more quickly in the same distance.

Table 20. DBEV Fleet size for different velocities in the dispatching market

	Switching stations					Recharging stations				
v [km/h]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]
25.0	21.9	836	105	12	953	21.9	836	105	34	975
		88%	11%	1%	100%		86%	11%	3%	100%
30.0	18.4	585	88	9	682	18.4	585	88	28	701
		86%	13%	1%	100%		83%	13%	4%	100%
35.0	15.9	434	76	7	517	15.9	434	76	24	534
		84%	15%	1%	100%		81%	14%	4%	100%

When the hourly cost, C_h, decreases, the taxi supply per hour and area of service, λ_d, increases and the percentage weight of the taxis carrying a passenger increases smoothly as it is observed in table (21). Thus, there is a proportional increasing of the number of taxis with the decreasing of the hourly cost.

Table 21. DBEV Fleet size for different hourly in the dispatching market

	Switching stations					Recharging stations				
C_h [€/h]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]
20	21.79	832	104	12	948	21.79	832	104	34	970
		88%	11%	1%	100%		86%	11%	4%	100%
17.5	21.93	837	105	12	954	21.93	837	105	34	976
		88%	11%	1%	100%		86%	11%	3%	100%
15.0	22.1	843.0	106.0	12.0	961.0	22.1	843.0	106.0	34.0	983.0
		88%	11%	1%	100%		86%	11%	3%	100%

4.3.2.2. Hailing market for DBEV

As observed in figure (28) the unitary cost for the hailing market is plot for a velocity of 25, 30 and 35 m/s with a constant city area, A, and demand for taxi trips per hour and area, λ_u. It can be seen how the reduction of the velocity, v, reduces also the unitary cost and at the same time decreases the minimum taxi supply, λ_u, associated. There is also an asymptote when the taxi hourly supply per hour and area of service equals the trip demand per hour and area of service multiplied for the time servicing a trip. This asymptote is about 19 taxis per hour and area of service for a velocity of 25 m/s, 14.5 for a velocity of 30 m/s and 13 for a velocity of 35 m/s. Therefore there is an exponential decreasing of the hourly taxi supply per area of service when velocity increases. Right after the minimum supply, all curves grow with the same slope given that a velocity change modifies de user cost and not the taxi one.

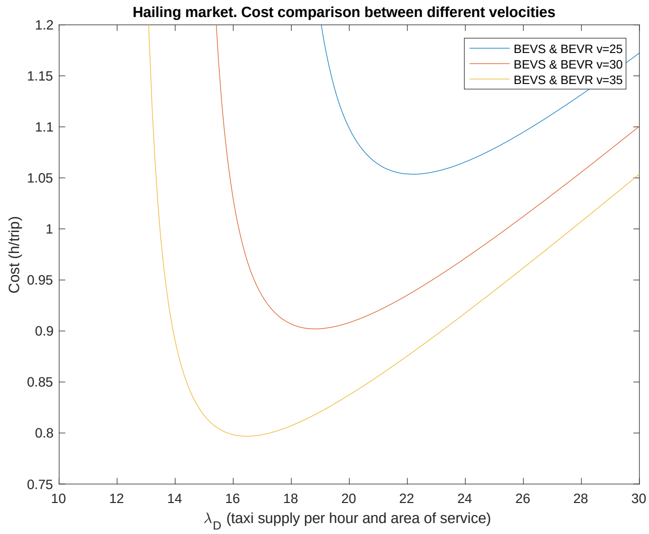


Figure 28. DBEV cost comparison between different velocities in the hailing market

The minimum taxi supply obtained either for vehicles that works with a switching or recharging station are equals as it is observed in the table (22). When the velocity is 25 m/s the user costs represents the 91-92% of the total one, while when the velocity reaches 35 m/s this percentage decreases to 85-86%. Moreover, even though the taxi cost percentage weight is not that relevant as other markets, with a 2-4% for 25 m/s, it increases to 9-10% for a velocity of 35 m/s. The infrastructure cost is higher for switching stations than for the recharging ones and the percentage weight barely reach the 0% for both of them. What is especially relevant for the hailing market is the cost of the city, what represents the 6% for a velocity of 25% and barely changes for velocity of 35 m/s. This cost for the city is due because the congestion that produces and if the velocity increases congestions will be technically reduced. Besides, we can see how for the BEVS the total cost is somewhat a 0.0010 h lower than the BEVR because of the infrastructure cost, what in practice means about 3.6 seconds cheaper.

Table 22. DBEV cost comparison between different velocities in the hailing market

Switching stations						
v [km/h]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
			0.99212	0.02376	0.00121	0.0669
25	22.3	1.05361	92%	2%	0%	6%
			0.90978	-0.0466	0.0012	0.0567
30	18.9	0.9020	91%	5%	0%	6%
			0.85321	-0.0941	0.00119	0.0498
35	16.6	0.79685	85%	10%	0%	5%

Recharging stations						
v [km/h]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
25	22.6	1.05264	0.9692	0.0299	0.0004	0.0678
			91%	4%	0%	6%
30	19.2	0.9010	0.89309	-0.0405	0.00039	0.0576
			90%	4%	0%	6%
35	16.8	0.7960	0.8422	-0.0902	0.0004	0.0504
			86%	9%	0%	5%

The next plot shows the variation of the unitary cost due to the taxi supply per hour and area of service for different hourly costs, C_h=20, 17.5 and 15, with a constant hourly demand per hour and area, λ_u, city area, A, and velocity, v, of 25 m/s. As it is possible to appreciate, the less hourly cost needed, the less unitary cost but the more taxi supply per hour and area of service needed, λ_d. This is because a reduction in the hourly cost implies a reduction of the taxi cost while other agents remain and a new equilibrium is found what requires more taxi supply but not necessarily more vehicles as it will be analyzed immediately thereafter. As it has happened before, there is an asymptote at 17.5 what remains for all the equations due to there is no change in the user cost. Right after the minimum supply per hour and area of service different slopes grow given that an hourly cost change modifies the user cost slope. The difference of distance between the approximately straight curves is due because of the cut-off point of the taxi cost function with the ordinates axis what is proportional with the hourly cost, C_h.

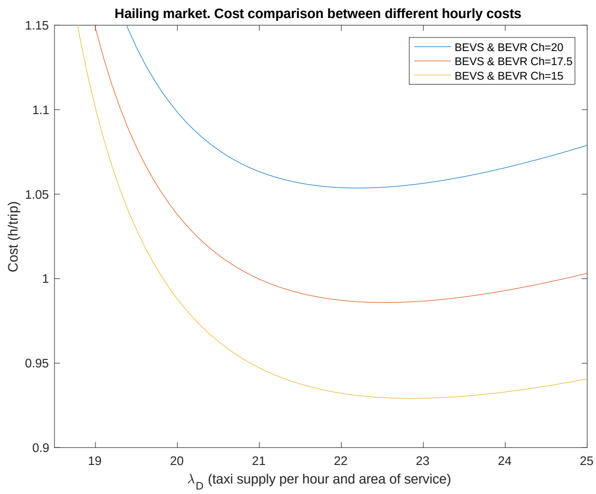


Figure 29. DBEV Cost comparison between different hourly costs in the hailing market

The minimum taxi supply per hour and area obtained either for vehicles that works with a switching or recharging station are higher for BEVR as it is observed in the following table (23). In this case, the taxi supply found is higher when the hourly cost decreases. Unlike other cases, a change in the hourly cost affects the percentage weight between de user, the taxi, the infrastructure cost and the city cost. When the hourly cost decreases, the percentage weight changes in the same way when velocity increases: user and city cost decrease, while taxi cost increases. The user cost represents 91-92% when C_h is 20 €/h and decreases to the 86% when C_h is 15 €/h, also the cost for the city remains in the 6-7%.

Table 23. DBEV Cost comparison between different hourly costs in the hailing market

Switching stations						
C_h [€/h]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
			0.9921	0.0238	0.0012	0.0669
20.0	22.3	1.0536	92%	2%	0%	6%
			0.9819	-0.0382	0.0012	0.0678
17.5	22.6	0.9859	90%	4%	0%	6%
			0.9704	-0.0882	0.0012	0.0690
15.0	23.0	0.9292	86%	8%	0%	6%
Recharging stations						
C_h [€/h]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
			0.9692	0.0299	0.0004	0.0678
20.0	22.6	1.0526	91%	3%	0%	6%
			0.9595	-0.0311	0.0004	0.0690
17.5	23.0	0.9847	90%	3%	0%	7%
			0.9512	-0.0822	0.0004	0.0702
15.0	23.4	0.9281	86%	7%	0%	6%

There is an important decreasing of the amount of taxis needed when velocity is reduced both for BEVS and BEVR, from a velocity of 25 m/s to 30 m/s, the fleet needed is reduced a 28% both the BEVS and BEVR, while from a 30 m/s to 35 m/s, the reduction is the 22% also for BEVS and BEVR, as it is observed in the table (24). Therefore, there is an exponential decreasing of the number of taxis with the velocity. On the other hand, the number of vehicles in the charging state is higher for the BEVR than for the BEVS because the time in the charging state for BEVS is 0.37 h lower. Likewise, a reduction in the velocity, gives a higher amount of taxis in the empty state, decreasing the ones carrying a passenger. This is due because the taxi trip will be more quickly in the same distance.

Table 24. DBEV Fleet size for different velocities in the hailing market

	Switching stations					Recharging stations				
v [km/h]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]
		774	222	11	1007		784	238	32	1054
25	22.3	77%	22%	1%	100%	22.6	74%	23%	3%	100%
		547	169	8	724		555	183	27	765
30	18.9	76%	23%	1%	100%	19.2	73%	24%	4%	100%
		412	140	7	559		417	149	23	589
35	16.6	74%	25%	1%	100%	16.8	71%	25%	4%	100%

When the hourly cost, C_h, decreases, the taxi supply per hour and area of service, λ_d, increases and the percentage weight of the taxis carrying a passenger increases smoothly as it is observed in table (25). Thus, there is a proportional increasing of the number of taxis with the decreasing of the hourly cost.

Table 25. DBEV Fleet size for different hourly costs in the hailing market

	Switching stations					Recharging stations				
C_h [€/h]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]
20	22.3	774	222	11	1007	22.6	784	238	32	1054
		77%	22%	1%	100%		74%	23%	3%	100%
17.5	22.6	784	238	11	1033	23	798	261	33	1092
		76%	23%	1%	100%		73%	24%	3%	100%
15	23	798	261	11	1070	23.4	812	284	33	1129
		75%	24%	1%	100%		72%	25%	3%	100%

4.3.2.3. Stand market for DBEV

In the next figure the unitary cost for the dispatching market is plot for a velocity of 25, 30 and 35 m/s with a constant city area, A, and demand for taxi trips per hour and area, λ_u. It can be seen how the reduction of the velocity, v, reduces also the unitary cost and at the same time decreases the minimum taxi supply associated, λ_d. There is also an asymptote when the taxi hourly supply per hour and area of service equals the trip demand per hour and hour of service multiplied for the servicing trip time, this makes the access time denominator zero and therefore an indeterminate in the user cost equation. This asymptote is about 18.4 taxis per hour and area of service for a velocity of 25 m/s, 15.8 for a velocity of 30 m/s and 13.5 for a velocity of 35 m/s. On account on that fact, there is an exponential decreasing of the hourly taxi supply per area of service when velocity increases. Right after the minimum supply, all curves grow with the same slope given that a velocity change modifies de user cost and not the taxi one.

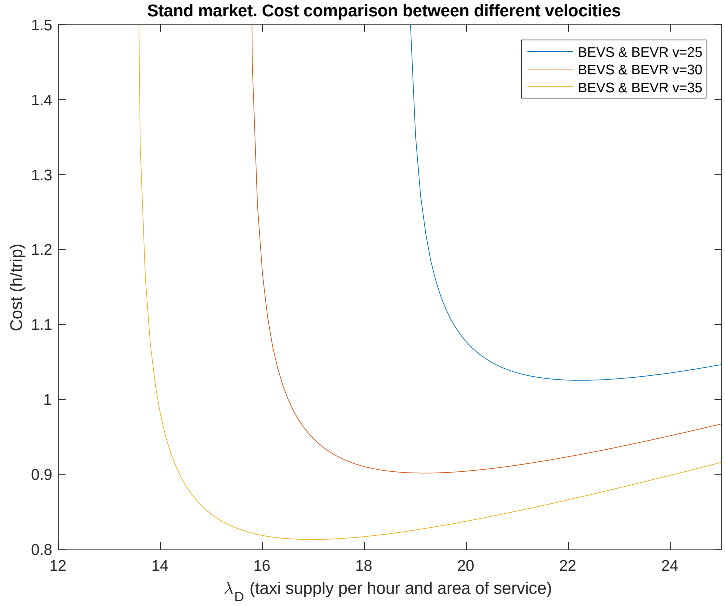


Figure 30. DBEV cost comparison between different velocities in the stand market

The minimum taxi supply obtained either for vehicles that works with a switching or recharging station are equal as we can check in the following table (26). When the velocity is 25 m/s the user costs represents the 98% of the total one, while when the velocity reaches 35 m/s this percentage decreases to 91%. Whereas, the taxi cost percentage weight increases from the 2% to 9%. The infrastructure cost is higher for switching stations than for the recharging ones but the percentage weight barely reach the 0% for both cases.

Table 26. DBEV cost comparison between different velocities in the stand market

Switching stations						
v [km/h]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
			1.0019	0.0205	0.0022	0.0000
25	22.3	1.0254	98%	2%	0%	0%
			0.93979	-0.0411	0.00221	0
30	19.3	0.9017	96%	4%	0%	0%
			0.8963	-0.0862	0.0022	0.0000
35	17.1	0.8130	91%	9%	0%	0%
Recharging stations						
v [km/h]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
			1.0019	0.0201	0.0018	0.0000
25	22.3	1.0246	98%	2%	0%	0%
			0.93979	-0.0415	0.00182	0
30	19.3	0.9008	96%	4%	0%	0%
			0.8963	-0.0867	0.0018	0.0000
35	17.1	0.8122	91%	9%	0%	0%

The next plot (31) shows the variation of the unitary cost due to the taxi supply per hour and area of service, λ_d, for different hourly costs, C_h=20, 17.5 and 15, with a constant hourly demand per hour and area, λ_u, city area, A, and velocity, v, of 25 m/s. As it is possible to appreciate, the less hourly cost needed, the less unitary cost but the more taxi supply per hour and area of service needed. This is because a reduction in the hourly cost implies a reduction of the taxi cost while other agents remain and a new equilibrium is found what requires more taxi supply but not necessarily more vehicles as it will be analyzed immediately thereafter. As it has happened before, there is an asymptote at 19 what remains for all the equations due to there is no change in the user cost. Right after the minimum supply per hour and area of service different slopes grow given that an hourly cost change modifies the user cost slope. The difference of distance between the approximately straight curves is due because of the cut-off point of the taxi cost function with the ordinates axis what is proportional with the hourly cost, C_h.

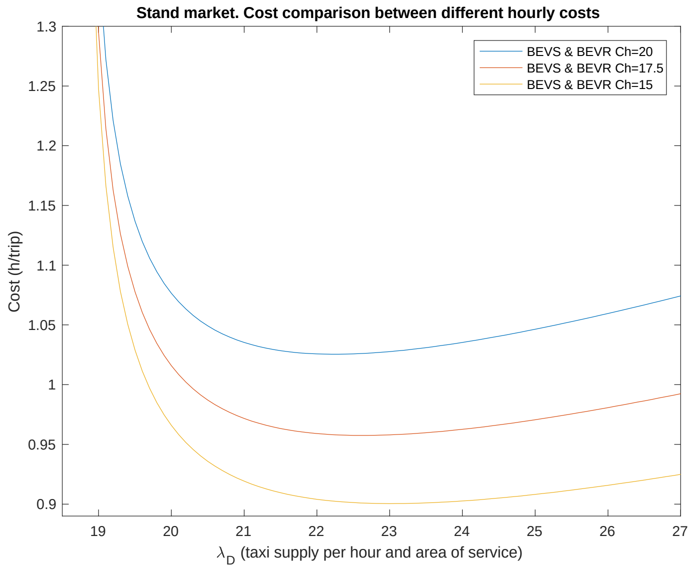


Figure 31. DBEV Cost comparison between different hourly costs in the stand market

The minimum taxi supply per hour and area obtained either for vehicles that works with a switching or recharging station are equal as we can check in the following table (27). In this case, the taxi supply found is higher when the hourly cost decreases. Unlike other cases, a change in the hourly cost affects the percentage weight between de user, the taxi and the infrastructure cost. When the hourly cost decreases, the percentage weight changes in the same way when velocity increases: user cost decrease, while taxi cost increases. In this market, when the hourly cost decreases, the taxi supply per hour and area of service increases from 22.3 to 22.7 when Ch goes from 20 €/h to 17.5 €/h, and the same increasing from 17.5 €/h to 15 €/h, that goes from 22.7 to 23.1 taxis per hour and area of service. The main cost at this point is because of the user that represents the 98% when Ch is 20 €/h and goes until 91 when Ch is 15 €/h. This 7% of difference is absorbed by the taxi cost going from the 2% to the 9%. This is because the user cost decreases while the taxi supply increases due to the decreasing exponential access time to the stand equation.

Table 27. DBEV Cost comparison between different hourly costs in the stand market

Switching stations						
C_h [€/h]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
			1.0019	0.0205	0.0022	0.0000
20.0	22.3	1.0254	98%	2%	0%	0%
			0.9944	-0.0401	0.0024	0.0000
17.5	22.7	0.9575	96%	4%	0%	0%
			0.9879	-0.0908	0.0026	0.0000
15.0	23.1	0.9005	91%	9%	0%	0%
Recharging stations						
C_h [€/h]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
			1.0019	0.0201	0.0018	0.0000
20.0	22.3	1.0246	98%	2%	0%	0%

			0.9944	-0.0405	0.0020	0.0000
17.5	22.7	0.9567	96%	4%	0%	0%
			0.9879	-0.0912	0.0022	0.0000
15.0	23.1	0.8996	91%	9%	0%	0%

There is an important decreasing of the amount of taxis needed when velocity is reduced both for BEVS and BEVR, from a velocity of 25 m/s to 30 m/s, the fleet needed is reduced a 26% both the BEVS and BEVR, while from a 30 m/s to 35 m/s, the reduction is the 22% also for BEVS and BEVR, as we can see in the table (28). Therefore, there is an exponential decreasing of the number of taxis with the velocity. On the other hand, the number of vehicles in the charging state is higher for the BEVR than for the BEVS because the time in the charging state for BEVS is 0.37 h lower. Likewise, a reduction in the velocity, gives a higher amount of taxis in the empty state, decreasing the ones carrying a passenger. This is due because the taxi trip will be more quickly in the same distance.

Table 28. DBEV Fleet size for different velocities in the stand market

	Switching stations					Recharging stations				
v [km/h]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]
		813	97	11	921		813	97	33	943
25	22.3	88%	11%	1%	100%	22.3	86%	10%	3%	100%
		586	84	9	679		586	84	28	698
30	19.3	86%	12%	1%	100%	19.3	84%	12%	4%	100%
		445	75	7	527		445	75	25	545
35	17.1	84%	14%	1%	100%	17.1	82%	14%	5%	100%

When the hourly cost, C_h, decreases, the taxi supply per hour and area of service, λ_d, increases and the percentage weight of the taxis carrying a passenger increases smoothly as it is observed in table (29). Thus, there is a proportional increasing of the number of taxis with the decreasing of the hourly cost.

Table 29. DBEV Fleet size for different hourly costs in the stand market

	Switching stations					Recharging stations				
C_h [€/h]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]
		813	97	11	921		813	97	33	943
20	22.3	88%	11%	1%	100%	22.3	86%	10%	3%	100%
		827	99	11	937		827	99	34	960
17.5	22.7	88%	11%	1%	100%	22.7	86%	10%	4%	100%
		842	101	12	955		842	101	34	977
15	23.1	88%	11%	1%	100%	23.1	86%	10%	3%	100%

4.3.2.4. Dispatching-hailing market for DBEV

The dispatching-hailing market is a mix between both markets and therefore will have both features related with these markets. As observed in figure (32) the unitary cost for the dispatching market is plot for a velocity of 25, 30 and 35 m/s with a constant city area, A, and demand for taxi trips per hour and area, λ_u. It can be seen how the reduction of the velocity reduces also the unitary cost and at the same time decreases the minimum taxi supply associated. There is also an asymptote when the taxi hourly supply per hour and area of service equals the trip demand per hour and area of service multiplied for the time. This asymptote is about 20 taxis per hour and area of service for a velocity of 25 m/s, 16 for a velocity of 30 m/s and 14 for a velocity of 35 m/s. We can therefore state there is an exponential decreasing of the hourly taxi supply per area of service when velocity increase. Right after the minimum supply, all curves grow with the same slope given that a velocity change modifies de user cost and not the taxi one.

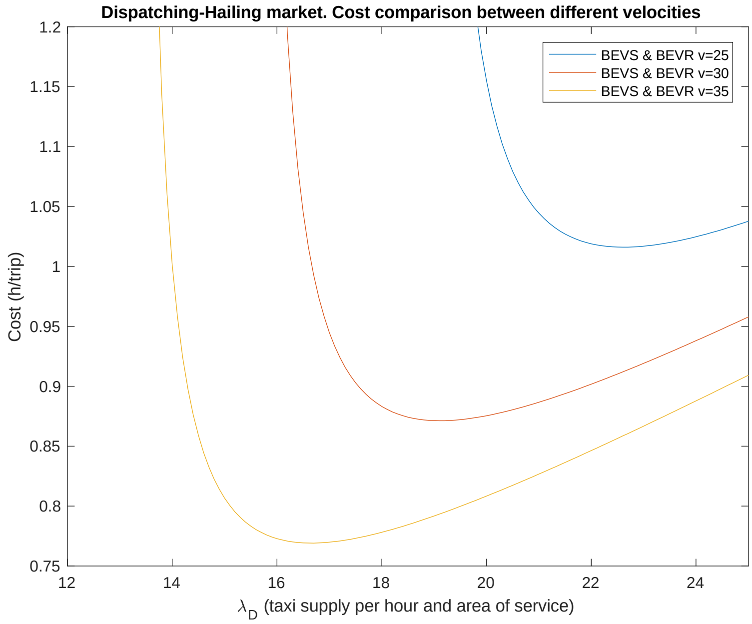


Figure 32. DBEV cost comparison between different velocities in the dispatching-hailing market

The minimum taxi supply obtained either for vehicles that works with a switching or recharging station are equals as we can check in the following table (30). Owing to this market is a mixture between the dispatching and the hailing ones, it will have values between both of them. When the velocity is 25 m/s the taxi supply is 10.9, when the velocity is 30 m/s the taxi supply is 9.5 and reaches the value of 8.5 when velocity goes at 35 m/s. On the other hand, when the velocity is 25 m/s the user costs represents the 88% of the total one, while when the velocity reaches 35 m/s this percentage decreases to 82%. The taxi cost percentage weight represents the 3% for a vekocity of 25 m/s, about the 0% for a velocity of 30 m/s and the 2% for a velocity of 35 m/sThe infrastructure cost is higher for switching stations than for the recharging ones and the percentage weight barely reach the 0% for both of them. The minim taxi supply decreases exponentially. Besides, we can see how for the BEVS the total cost is somewhat a 0.0008 h lower than the BEVR because of the infrastructure cost, what in practice means about 2 seconds cheaper.

Table 30. DBEV cost comparison between different velocities in the dispatching-hailing market

Switching stations						
v [km/h]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
			0.9204	-0.1383	0.0000	0.0687
25	22.9	1.0892	82%	12%	0%	6%

30	19.4	0.9523	0.8583	-0.1539	0.0000	0.0582
			76%	14%	0%	5%
35	16.9	0.8558	0.8147	-0.1663	0.0000	0.0507
			72%	15%	0%	4%
Recharging stations						
v [km/h]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
25	22.7	1.0161	0.8941	-0.1850	0.0013	0.0681
			79%	16%	0%	6%
30	19.2	0.8712	0.8369	-0.2134	0.0013	0.0576
			74%	19%	0%	5%
35	16.8	0.7691	0.7953	-0.2334	0.0013	0.0504
			71%	21%	0%	4%

The next plot shows the variation of the unitary cost due to the taxi supply per hour and area of service for different hourly costs, C_h=20, 17.5 and 15, with a constant hourly demand per hour and area, λ_u, city area, A, and velocity, v, of 25 m/s. As it is possible to appreciate, the less hourly cost needed, the less unitary cost but the more taxi supply per hour and area of service needed, λ_d. This is because a reduction in the hourly cost implies a reduction of the taxi cost while other agents remain and a new equilibrium is found what requires more taxi supply but not necessarily more vehicles as it will be analyzed immediately thereafter. As it has happened before, there is an asymptote at 20.6 what remains for all the equations due to there is no change in the user cost. Right after the minimum supply per hour and area of service different slopes grow given that an hourly cost change modifies the user cost slope. The difference of distance between the approximately straight curves is due because of the cut-off point of the taxi cost function with the ordinates axis what is proportional with the hourly cost, C_h.

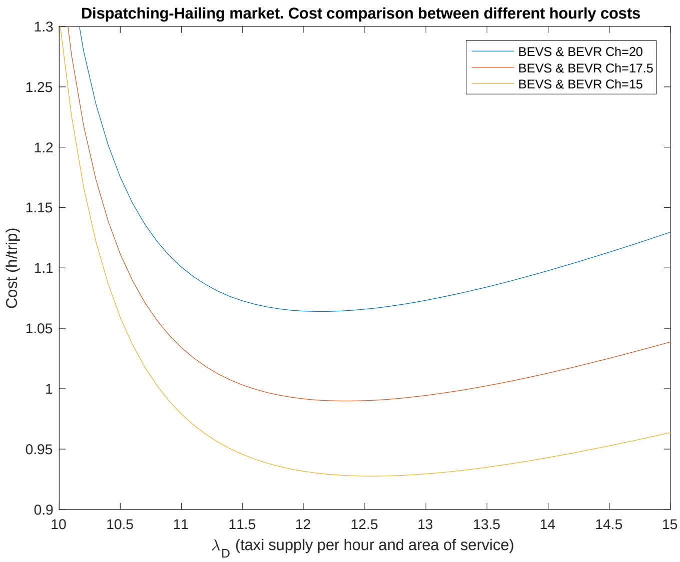


Figure 33. DBEV Cost comparison between different hourly costs in the dispatching-hailing market

We can see as it happens in the other markets how when the taxi supply increases, the taxi cost weight percentage absorbs the user one and this is because when the taxi supply increases, the user cost decreases exponentially, as well as, the city cost, whereas the taxi cost increases linearly. What is relevant in this market is whereas in the hailing market the city cost percentage weight decreases, in the dispatching-hailing remains. We have seen how the percentage weight remained at the dispatching market when the taxi supply per hour and area of service increased because of the reduction of the hourly cost. In this market, when the hourly cost decreases, the taxi supply per hour and area of service increases from 10.9 to 11.1 when Ch goes from 20 €/h to 17.5 €/h, and the same increasing from 17.5 €/h to 15 €/h, that goes from 11.1 to 11.3 taxis per hour and area of service. The user cost represents the 88% when Ch is 20 €/h and goes until 91% when Ch is 15 €/h.

Table 31. DBEV Cost comparison between different hourly costs in the dispatching-hailing market

Switching stations						
C_h [€/h]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
20.0	12.2	1.0640	0.9182	-0.0251	0.0025	0.0366
			93%	3%	0%	4%
17.5	12.4	0.9898	0.9138	-0.0965	0.0025	0.0372
			87%	9%	0%	4%
15.0	12.7	0.9276	0.9081	-0.1546	0.0025	0.0381
			82%	14%	0%	4%
Recharging stations						
C_h [€/h]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
20.0	12.3	1.1600	0.95422	0.03467	0.00025	0.0369
			93%	3%	0%	4%
17.5	12.5	1.0854	0.94749	-0.0348	0.00025	0.0375
			93%	3%	0%	4%
15.0	12.8	1.0227	0.93881	-0.0905	0.00025	0.0384
			88%	8%	0%	4%

There is an important decreasing of the amount of taxis needed when velocity is reduced both for BEVS and BEVR, from a velocity of 25 m/s to 30 m/s, the fleet needed is reduced a 28% both the BEVS and BEVR, while from a 30 m/s to 35 m/s, the reduction is the 24% also for BEVS and BEVR, as we can see in the table (32). Therefore, there is an exponential decreasing of the number of taxis with the velocity. On the other hand, the number of vehicles in the charging state is higher for the BEVR than for the BEVS because the time in the charging state for BEVS is 0.37 h lower. Likewise, a reduction in the velocity, gives a higher amount of taxis in the empty state, decreasing the ones carrying a passenger. This is due because the taxi trip will be more quickly in the same distance.

Table 32. DBEV Fleet size for different velocities in the dispatching-hailing market

	Switching stations					Recharging stations				
v [km/h]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]
25	22.9	795	216	36	1046	22.7	788	208	12	1007
		76%	21%	3%	100%		75%	20%	1%	100%
30	19.4	561	166	30	756	19.2	555	158	9	722
		74%	23%	3%	100%		77%	22%	1%	100%
35	16.9	419	133	26	577	16.8	417	130	7	553
		73%	24%	3%	100%		75%	24%	1%	100%

When the hourly cost, C_h, decreases, the taxi supply per hour and area of service, λ_d, increases and the percentage weight of the taxis carrying a passenger increases smoothly as it is observed in table (33). Thus, there is a proportional increasing of the number of taxis with the decreasing of the hourly cost.

Table 33. DBEV Fleet size for different hourly costs in the dispatching-hailing market

	Switching stations					Recharging stations				
C_h [€/h]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]
20	12.2	424	143	7	574	12.3	427	148	20	595
		74%	25%	1%	100%		72%	25%	3%	100%
17.5	12.4	424	153	7	584	12.5	434	158	21	613
		73%	26%	1%	100%		71%	26%	3%	100%
15	12.7	441	168	7	616	12.8	444	173	21	638
		72%	27%	1%	100%		70%	27%	3%	100%

4.3.2.5. Stand-hailing market for DBEV

The stand-hailing market is a mix between both markets and therefore will have both features related with these markets. In the next figure (34) the unitary cost for the dispatching market is plot for a velocity of 25, 30 and 35 m/s with a constant city area, A, and demand for taxi trips per hour and area, λ_u. It can be seen how the reduction of the velocity reduces also the unitary cost and at the same time decreases the minimum taxi supply associated. There is also an asymptote when the taxi hourly supply per hour and area of service equals the trip demand per hour and area of service multiplied for the time. This asymptote is about 17.5 taxis per hour and area of service for a velocity of 25 m/s, 15 for a velocity of 30 m/s and 6 for a velocity of 35 m/s. We can therefore state there is an exponential decreasing of the hourly taxi supply per area of service when velocity increases. Right after the minimum supply, all curves grow with the same slope given that a velocity change modifies de user cost and not the taxi one.

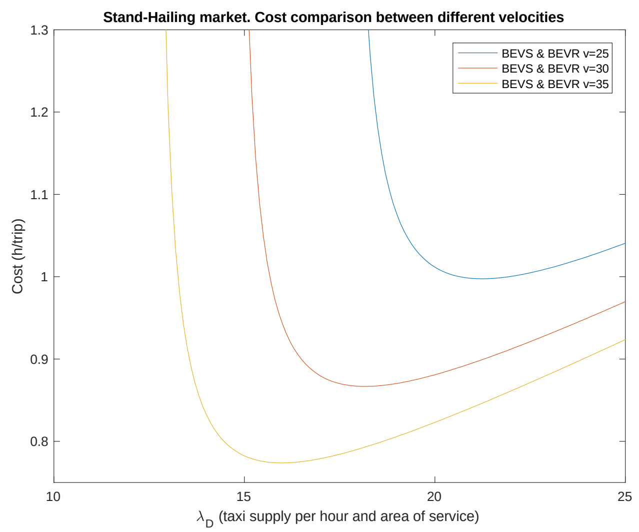


Figure 34. DBEV cost comparison between different velocities in the stand-hailing market

The minimum taxi supply obtained either for vehicles that works with a switching or recharging station are equals as we can check in the following table (34).

Table 34. DBEV cost comparison between different velocities in the stand-hailing market

Switching stations						
v [km/h]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
25	21.3	0.9975	0.9434	-0.0118	0.0019	0.0639
			92%	1%	0%	6%
30	18.3	0.86675	0.88097	-0.0711	0.00195	0.0549
			87%	7%	0%	6%
35	16.1	0.77381	0.8381	-0.1145	0.0020	0.0483
			84%	11%	0%	5%
Recharging stations						
v [km/h]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
25	21.3	0.99667	0.9434	-0.0115	0.0009	0.0639
			93%	1%	0%	6%
30	18.3	0.86594	0.88097	-0.0708	0.0009	0.0549
			87%	7%	0%	6%
35	16.1	0.773	0.8381	-0.1143	0.0009	0.0483
			84%	11%	0%	5%

The next plot shows the variation of the unitary cost due to the taxi supply per hour and area of service, λ_d, for different hourly costs, C_h=20, 17.5 and 15, with a constant hourly demand per hour and area, λ_u, city area, A, and velocity, v, of 25 m/s. As it is possible to appreciate, the less hourly cost needed, the less unitary cost but the more taxi supply per hour and area of service needed. This is because a reduction in the hourly cost implies a reduction of the taxi cost while other agents remain and a new equilibrium is found what requires more taxi supply but not necessarily more vehicles as it will be analyzed immediately thereafter. As it has happened before, there is an asymptote at 18 what remains for all the equations due to there is no change in the user cost. Right after the minimum supply per hour and area of service different slopes grow given that an hourly cost change modifies the user cost slope. The difference of distance between the approximately straight curves is due because of the cut-off point of the taxi cost function with the ordinates axis what is proportional with the hourly cost, C_h.

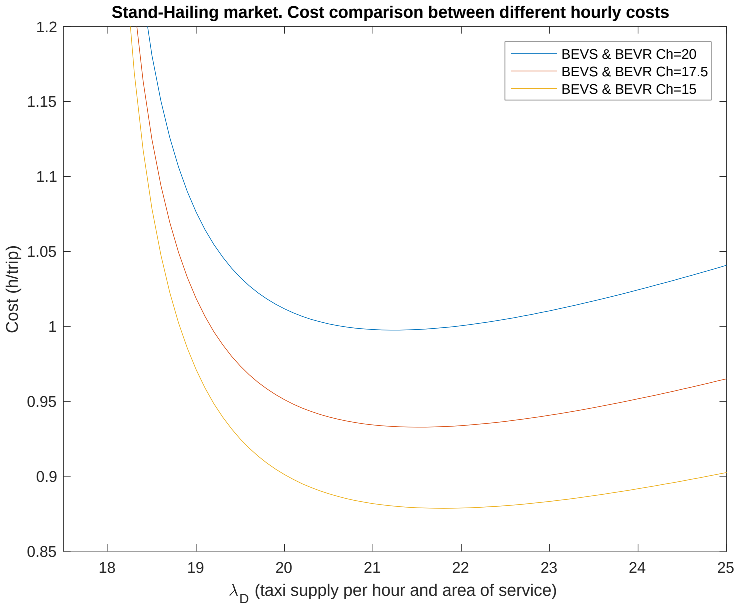


Figure 35. Cost comparison between different hourly costs in the stand-hailing market

The minimum taxi supply per hour and area obtained either for vehicles that works with a switching or recharging station are equal as we can check in the following table (35).

Table 35. DBEV Cost comparison between different hourly costs in the stand-hailing market

Switching stations						
C_h [€/h]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
			0.94345	-0.0118	0.00192	0.0639
20.0	21.3	0.9975	92%	1%	0%	6%
			0.9371	-0.0712	0.00198	0.0648
17.5	21.6	0.9327	87%	7%	0%	6%
			0.93159	-0.1207	0.00204	0.0657
15.0	21.9	0.8786	83%	11%	0%	6%
Recharging stations						
C_h [€/h]	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
			0.94345	-0.0115	0.00086	0.0639
20.0	21.3	0.9967	93%	1%	0%	6%
			0.9371	-0.0709	0.00092	0.0648
17.5	21.6	0.9319	87%	7%	0%	6%
			0.93159	-0.1205	0.00098	0.0657
15.0	21.9	0.8778	83%	11%	0%	6%

There is an important decreasing of the amount of taxis needed when velocity is reduced both for BEVS and BEVR, from a velocity of 25 m/s to 30 m/s, the fleet needed is reduced a 27% both the BEVS, while from a 30 m/s to 35 m/s, the reduction is the 23% also for BEVS, as we can see in the table (36). Therefore, there is an exponential decreasing, both for BEVS and BEVR, of the number of taxis with the velocity. On the other hand, the number of vehicles in the charging state is higher and the ones carrying a passenger lower for the BEVR than for the BEVS because the time in the charging state for BEVS is 0.37 h lower. Likewise, a reduction in the velocity, gives a higher amount of taxis in the empty state, decreasing the ones carrying a passenger. This is due because the taxi trip will be more quickly in the same distance.

Table 36. DBEV Fleet size for different velocities in the stand-hailing market

	Switching stations					Recharging stations				
v [km/h]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]
		739	150	11	900		739	150	33	922
25	21.3	82%	17%	1%	100%	21.3	80%	16%	4%	100%
		529	126	9	664		529	126	27	682
30	18.3	80%	19%	1%	100%	18.3	78%	18%	4%	100%
		399	108	7	514		399	108	24	531
35	16.1	78%	21%	1%	100%	16.1	75%	20%	5%	100%

When the hourly cost, C_h, decreases, the taxi supply per hour and area of service, λ_d, increases and the percentage weight of the taxis carrying a passenger increases smoothly as it is observed in table (37). Thus, there is a proportional increasing of the number of taxis with the decreasing of the hourly cost.

Table 37. DBEV Fleet size for different hourly costs in the stand-hailing market

	Switching stations					Recharging stations				
C_h [€/h]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]
		739	150	11	900		739	150	33	922
20	21.3	82%	17%	1%	100%	21.3	80%	16%	4%	100%
		750	162	11	923		750	162	33	945
17.5	21.6	81%	18%	1%	100%	21.6	79%	17%	3%	100%
		760	174	12	946		760	174	34	968
15	21.9	80%	18%	1%	100%	21.9	79%	18%	4%	100%

4.3.3. DBEV: Comparison between markets

The analysis conducted below is a comparison of a hypothetic DBEV cars that have increased his velocity to 30 m/s and reduced the hourly cost to 17.5 €/h. The ultimate objective is to compare this driverless taxi between the different markets.

The analysis conducted below is a comparison between different markets studied above with the same trip demand, $\lambda_u = 50$. Once it has been seen there is no major difference between the BEVS and BEVR, it will be useful for the ease of the evaluation to plot these different markets just for BEVS.

As observed in the next figure (36), it can be seen the behavior of the unitary cost according to the taxi supply per hour and area of service, λ_d. It is interesting to appreciate that a lower cost does not mean a lower taxi supply per hour and area of service. For example, a driverless taxi in the stand-hailing market will have a lower minimum unitary cost than the stand market but will need a higher taxi supply for it. In case the taxi supply per hour and area of service is already known, the lowest unitary system cost will be provided by the dispatching market from 14.8 and for prior values: the stand-hailing market. In case of a city that already have the fleet size and bearing this in mind cities with a high taxi supply the dispatching mode will be the ideal, however, for cities with a low rate of taxi supply the stand-hailing will be the best. Besides, in cities with a certain value of λ_d some markets mill not be possible.

As it has been mentioned before, the lowest minimum unitary cost, will be provided by the dispatching market, whereas, the dispatching-hailing market will have the highest. On the other hand, the dispatching and hailing market will have the lowest taxi supply per hour and area of service, λ_d, and the stand market the highest.

In the foregoing sections it has been explained how the unitary cost varies over the trip demand per hour and area of service, λ_u, if this cost decreases or increases when that demand increases and vice versa. In case λ_u increases, the dispatching and the stand will decrease, however the hailing, the dispatching-hailing and the stand-hailing will increase, what means that for cities with a high λ_u the stand market will be the most optimal market for low λ_d and the dispatching one will be the best for high λ_d.

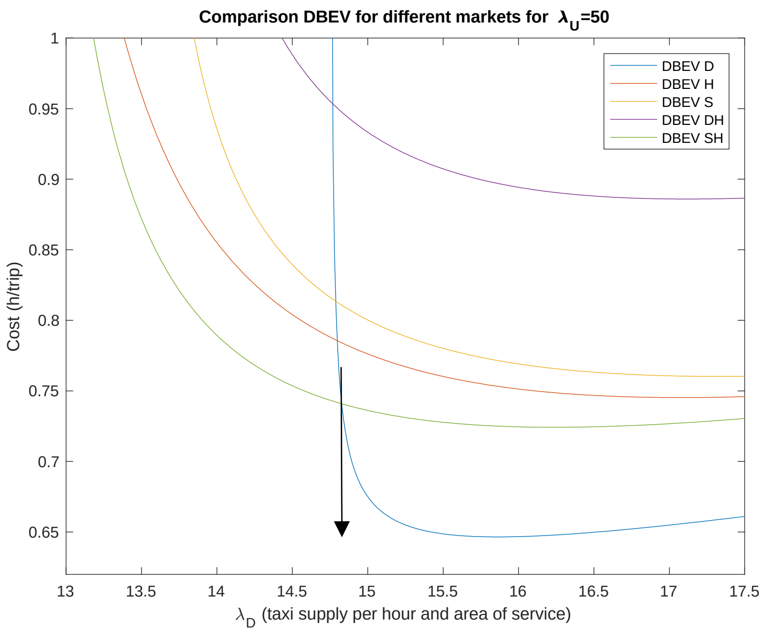


Figure 36. DBEV cost comparison between different markets

The following table (38) represents the cost of the different markets. The minimum taxi supply obtained either for vehicles that works with a switching or recharging station are equal and higher than the SV ones. The stand market is the one that has the highest user cost percentage weight, whereas the stand-hailing has the lowest, what means that the stand market with a combination of the hailing one can have the lowest percentage weight because of the introduction of the city cost. The stand-hailing market has the highest taxi unitary cost, while the hailing market has the lowest. The stand-hailing market will have the highest cost for the city.

In terms of the infrastructure cost, this is higher for the dispatching-hailing than the dispatching market, it is because inside the infrastructure equation both for switching or recharging, the number of charging station ports, C, are higher due to it is related to the total distance, which will be higher for the dispatching-hailing than the dispatching. For the stand-hailing market, the stand infrastructure cost will be added and therefore the total cost will be the highest one among all the markets.

Table 38. DBEV cost comparison between different markets

	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
	Switching stations					
D	16.0	0.6474	0.8010	-0.1547	0.0012	0.0000
			84%	16%	0%	0%
H	16.8	0.7466	0.8476	-0.1407	0.0012	0.0504
			82%	14%	0%	5%
S	17.4	0.7611	0.8907	-0.1328	0.0024	0.0000
			87%	13%	0%	0%
DH	17.0	0.7993	0.8738	-0.2845	0.0013	0.0510
			72%	24%	0%	4%
SH	16.3	0.7250	0.8339	-0.1598	0.0020	0.0489
			80%	15%	0%	5%
	Recharging stations					
	λ_d [taxis/h·A]	z [€]	z_u [€]	z_t [€]	z_I [€]	z_c [€]
D	16.0	0.6465	0.8010	-0.1551	0.0004	0.0000
			84%	16%	0%	0%
H	17.2	0.7454	0.8335	-0.1336	0.0004	0.0516
			82%	13%	0%	5%
S	17.4	0.7602	0.8907	-0.1332	0.0020	0.0000
			87%	13%	0%	0%
DH	17.2	0.8853	0.8904	-0.2152	0.0001	0.0516
			77%	19%	0%	4%
SH	16.3	0.7242	0.8339	-0.1595	0.0010	0.0489
			80%	15%	0%	5%

The next table (39) will show the fleet size required for all the markets. The highest number of vehicles needed will be for the hailing market, with 573 for the DBEVS and 617 for the DBEVR, due to they will be either running all the time searching for a customer to be hailed, riding the customer or in the charging state. On the other hand, the dispatching market will have the minimum fleet size required, with 520 for the DBEVS and 537 for the DBEVR, to obtain the minimum unitary cost. In addition, the dispatching and the stand markets will have the higher number of taxis in service percentage weight, 84-85%, whereas the hailing market, with 26-27%, will show the highest number of vehicles in the empty state. The number of vehicles in the charging state is higher for the DBEVR than for the DBEVS because the time in the charging state for DBEVS is 0.37 h lower.

Table 39. DBEV fleet size comparison between markets

	Switching stations					Recharging stations				
	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]	λ_d [taxis/h·A]	M_Q [taxis]	M_E [taxis]	M_C [taxis]	M [taxis]
		436	77	7	520		436	77	24	537
D	16	84%	15%	1%	100%	16	81%	14%	4%	100%
		417	149	7	573		427	166	24	617
H	16.8	73%	26%	1%	100%	16.8	69%	27%	4%	100%
		453	76	7	536		453	76	25	554
S	17.4	85%	14%	1%	100%	17.4	82%	14%	5%	100%
		422	137	7	566		427	143	26	596
DH	17	75%	24%	1%	100%	17	72%	24%	4%	100%
		404	114	7	525		404	114	24	542
SH	16.3	77%	22%	1%	100%	16.3	75%	21%	4%	100%

For the ease understanding of what is has explained above, the next bar chart (37) represents the table above,

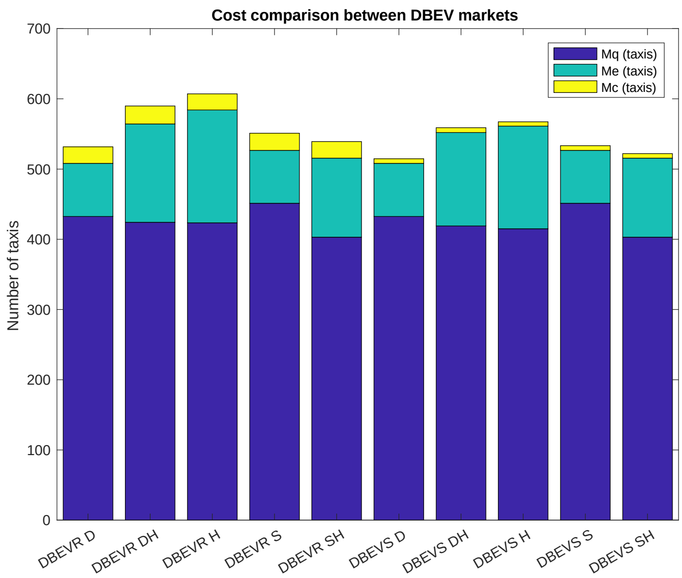


Figure 37. DBEV Bar chart of the fleet size comparison between markets

The following bar chart (38) shows the total cycle time per trip served by a taxi. In this case a number of key issues arise from this chart because results are different than for BEV vehicles. It has seen how the dispatching market was the highest

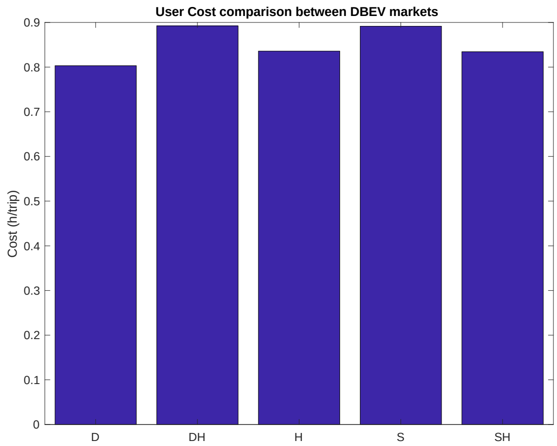


Figure 38. DBEV User cost comparison between markets

The next bar chart (39) shows how the stand-hailing market has the maximum taxi unitary cost when the total cost reaches the minimum, while the hailing one has the highest.

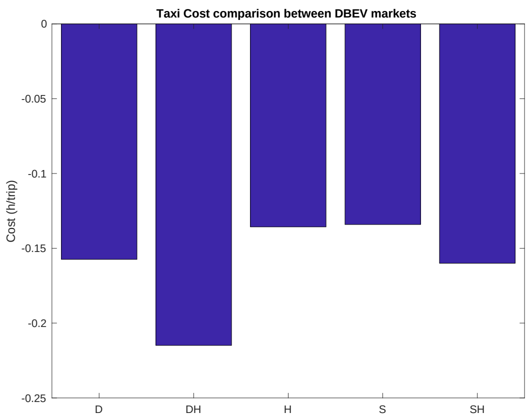


Figure 39. DBEV Taxi cost comparison between markets

The highest cycle distance is the dispatching one,

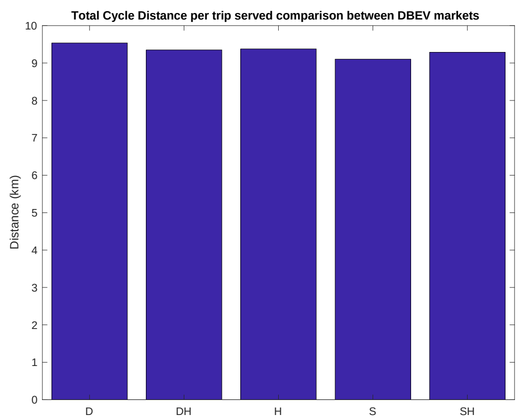


Figure 40. DBEV Total cycle distance per trip served comparison between markets

With regard to the riding time or total time obtained when the minimum unitary cost is gotten, the dispatching market shows the highest while the stand shows the shortest, due to in this market taxis will wait in a stand waiting for a customer.

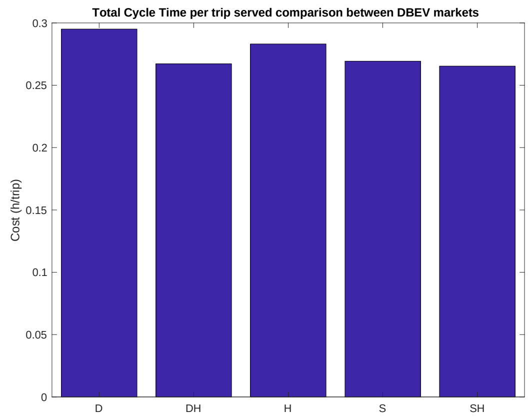


Figure 41. DBEV Total cycle time per trip served comparison between markets

The following bar chart (42) shows the total idle distance ridden by a trip. The stand market shows the lowest idle distance ridden being about less than the half of the dispatching market, due to on the stand market, taxis do not have to pick up the customer and the customer goes to the stand. The highest distances are showed in the stand-hailing followed by the dispatching-market. In these cases, distances are higher than the hailing market because in this model in these markets, the distance of a trip is the dispatching/stand trip plus the possible hailing one.

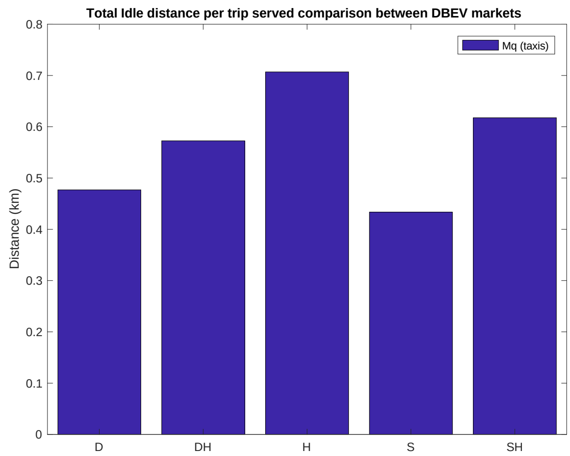


Figure 42. Total idle distance served comparison between DBEV markets

The user costs work as a basis for the total cost. As it is seen in the following figure (43), the asymptotes are because this cost.

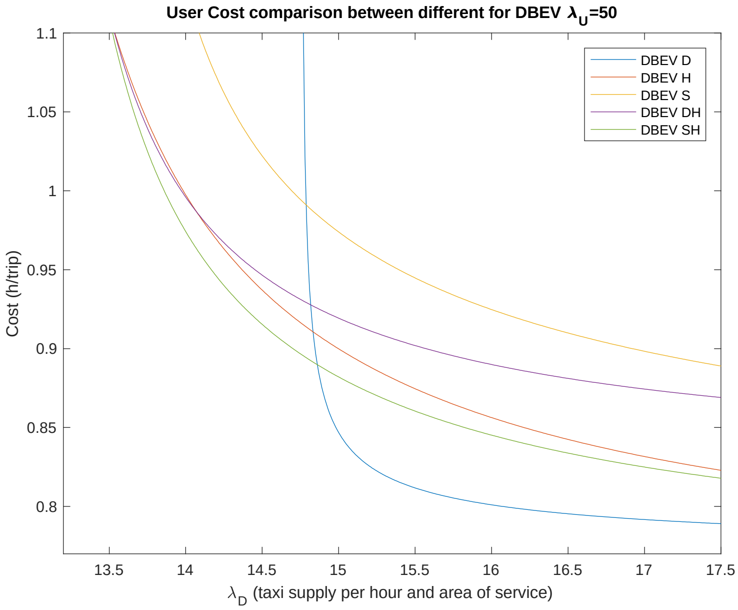


Figure 43. DBEV User cost comparison between different markets

The taxi cost are plot in the next figure (44),

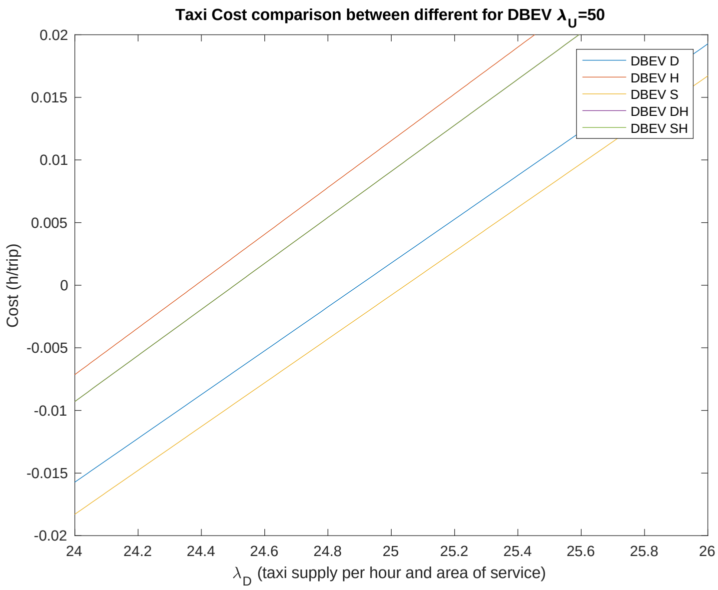


Figure 44. DBEV Taxi cost comparison between different markets

4.4. DISCUSSION OF RESULTS

Assuming there is a city of area A, with a value of time for the users VoT_u, where the hourly cost for taxis is C_{km} and their distance cost is C_{km}, the average fare as explained in the prior section is $\bar{c}$. Besides, in terms of externalities, the fuel consumption represents E_d, the emission of various pollutants F_c, the slope of the speed-density linear relation of the macroscopic diagram function is α and the average speed without the presence of taxis is v_1.

When the **trip demand, $\boldsymbol{\lambda_u}$, increases**,

- The taxis supply per hour and area of service, $\boldsymbol{\lambda_d}$**,** increases.
- The minimum cost z decreases for the dispatching and stand markets.
- The minimum cost z increases for the hailing market, dispatching-hailing and stand-hailing markets. Hence, whether hailing is involved in a market it will increase.
- The percentage weight of the user cost, z_u, of the minimum cost increases for the dispatching and the stand markets. However, this percentage decreases for the hailing, dispatching-hailing and stand-hailing markets.
- The percentage weight of the taxi cost, z_t, of the minimum cost increases for the dispatching-hailing and the stand-hailing markets. However, this percentage decreases for the dispatching, the hailing and the stand markets.
- The percentage weight of the external cost, z_C, of the minimum cost increases for the hailing and the stand-hailing markets, but remains for the dispatching-hailing market.
- There is no change in the percentage weight of the taxis for the dispatching, the stand and the stand-hailing markets. For M_Q increases in the dispatching and decreases in the dispatching-hailing. On the other hand, for M_E increases in the hailing and increases in the dispatching-hailing.

When **comparing SV and BEV**,

- The percentage weight of the user cost, z_u, is the strongest in the dispatching market and the weakest when hailing is introduced.
- The percentage weight of the taxi cost, z_t, is the strongest when the dispatching-hailing and stand-hailing markets, however, it is the weakest when it is purely hailing, being the external cost, z_C, the strongest in this last case.
- The percentage weights of the M_Q, M_E remain for all markets except for the hailing (M_Q increases and M_E decreases) and the dispatching-hailing (M_Q decreases and M_E increases).

When **increasing cruising velocity, $\boldsymbol{v}$**, because of the introduction of the DBEV,

- The taxis supply per hour and area of service, $\boldsymbol{\lambda_d}$**,** decreases.
- The percentage weight of the user cost, z_u, of the minimum cost decreases for all markets.
- The percentage weight of the taxi cost, z_t, of the minimum cost increases for all markets.
- The percentage weight of the external cost, z_C, of the minimum cost smoothly decreases for the dispatching market and remains for the hailing and stand-hailing ones.
- The percentage of the M_Q decreases, whilst M_E increases and M_C remains.

When **decreasing the hourly cost,** C_h, because of the introduction of the DBEV,

- The taxis supply per hour and area of service, $\boldsymbol{\lambda_d}$**,** increases.
- The percentage weight of the user cost, z_u, of the minimum cost increases for all markets.
- The percentage weight of the taxi cost, z_t, of the minimum cost decreases for all markets.
- The percentage weight of the external cost, z_C, of the minimum cost remains for the hailing, the dispatching-hailing and stand-hailing ones.
- The percentage of the M_Q decreases for the hailing, dispatching-hailing and stand-hailing markets while for the dispatching and stand remains.
- The percentage of the M_E increases for the hailing, dispatching-hailing and stand-hailing markets while for the dispatching and stand remains.
- The percentage of the M_C remains for all markets

When **comparing SV and BEV**,

- The percentage weight of the user cost, z_u, is the strongest in the stand market and the weakest when hailing is introduced as happened for SV and BEV.
- The percentage weight of the taxi cost, z_t, is the strongest when the dispatching-hailing and stand-hailing markets, however, it is the weakest when it is in a stand market. Besides, the percentage for the external cost, z_C, is equal for all markets with hailing involved.
- The percentage weights of the M_Q, M_E remain for all markets except for the hailing (M_Q increases and M_E decreases) and the dispatching-hailing (M_Q decreases and M_E increases).

CHAPTER 5

CONCLUSIONS

5.1. INTRODUCTION

In this last chapter, ***general*** and ***specific conclusions*** are presented as response of the objectives exposed at Chapter 1 emerging from the modelling design in chapter 3 and related analysis carried out at Chapter 4.

5.2. GENERAL CONCLUSIONS

The taxi modelling presented in this dissertation allows to obtain the performance of general and unitary cost per hour and area of service, λ_d, as well as other variables of the taxi systems in the dispatching, hailing, stand, dispatching-hailing and stand-hailing markets and with different types of taxi vehicles: SV, BEV and DBEV. This approach allows to compare in terms of either type of market or vehicle.

Furthermore, new markets defined in this study: the dispatching-hailing and the stand-hailing responds inside the range of dispatching, hailing and stand values obtained in the presented analysis and reviewed in other papers.

In addition, DBEV, responds successfully the expected results with the integrated modelling set out in Chapter 3: there is a reduction of the taxi system unitary cost when increasing the cruising velocity, $\bar{v}$, and decreasing the hourly cost, C_h.

5.3. SPECIFIC CONCLUSIONS

Once specific objectives are defined at Chapter 1 and the problem formulation and its analysis is carried out, several specific objectives are defined

For a city with a constant and medium-high trip demand and taxi supply ratio, λ_u/λ_d, the dispatching market has the best performance among the other ones for SV, BEV and DBEV taxi vehicles. Nevertheless, when this ratio is low, the stand-hailing market behaves better.

The sensibility of the trip demands responds different among all markets. For the dispatching and stand ones, whether the trip demand, λ_u, increases, the minimum unitary cost of the system will be reduced while its associated taxi supply, λ_d, becomes higher, however, for the hailing, the dispatching-hailing and stand-hailing markets, the minimum unitary cost achieved will be increased, as well as its taxi supply, λ_d, associated.

BEV performs better than the SV with lower minimum unitary cost. Key performance indicators related as it is the external cost for the city is decreased drastically since pollution factor is virtually removed. Nevertheless, longer distances are required to recharge the battery at the recharging or swapping stations what implies a slightly higher taxi cost. Besides, the minimum cost is obtained in the dispatching market.

The minimum taxi supply per hour and area of service, λ_d, in terms of SV taxis is obtained in the dispatching-hailing market what by means of the Little's Formula, has likewise the lowest fleet size, M. On the other hand, the maximum taxi supply, λ_d, among all the minimums (optimals) is found in the stand and dispatching markets with the highest fleet size, M. Likewise, the minimums for BEV taxis are for the standard-hailing and, unlike the SV, for the dispatching markets.

Moreover, DBEV presents benefits for all the system agents regarding the current type of vehicles mainly used nowadays for cities, the SV. These automated taxis allow to reduce the hourly cost, C_h, what brings a reduction in the taxi cost, z_t. Besides, thanks the new integrated taxi modelling distinguishes all types of distances and its associated velocities, it is possible to increase the ones that are directly affected as the in-vehicle, v_Q, the dispatching, v_D, and the charging, v_C, velocities and therefore, to obtain a reduction in the user cost, z_u. Hence, the reduction of these costs implies a drastically decreasing in the total taxi system cost and therefore, having a lower minimum cost than with SV and BEV taxis.

In terms of the taxi supply, λ_d, and the fleet size, M, the introduction of DBEV with switching stations provides the minimum values with the dispatching market and the highest minimums for the stand and hailing markets. With regarding the DBEV with recharging stations the lowest values of the taxi supply, λ_d, are provided by the dispatching market as it happens with the switching stations, however the minimum fleet size, M, is found in the stand-hailing. Likewise, the maximums are the same than the DBEV with switching stations.

Finally, taking into account these results respond to the city of **Barcelona**, it can be assumed DBEV, with medium-high λ_u/λ_d ratio, perform better for the dispatching market. Besides, for a low λ_u/λ_d ratio, the stand-hailing market have the best behaviour having the lowest cost, as it happened with SV and BEV taxis. In addition, DBEV might imply a reduction of the taxi fare for user, however, it depends on the supply and demand curve and its elasticity at it has been explained in prior sections: a reduction of the hourly cost might transform into holder license benefit and/or fare reduction. Hence, since Barcelona seems to increase its population in the coming future, a dispatching market would be the best option in a scenario where DBEV take place and get competitiveness facing the collective public transportation means and reducing externalities as pollution.

5.4. SUGGESTION FOR FURTHER RESEARCH

Despite the contributions described in the prior section, further research on the topics covered in this master's thesis is required. Hence, several suggestions for future research are proposed below.

This study has not taken into consideration the implementation of a taxi modelling that considers the dispatching-stand combination. It is interesting to have a wider range of options for the modelling.

With regarding how empty taxis are located within a station influence area once they have recharged, this this model has been developed for a uniformly distributed taxi availability distribution with high values of fleet size, however, there is a lack of other scenarios as when the density of available taxis decreases with the distance from stations but is uniform (intermediate fleet size values) (figure 4.b) and when the density of available taxis is decreasing (low fleet size values) (figure 4.c) estado del arte), as (Sathaye, 2014) suggests.

In the economic field, knowing the elasticity of these markets may solve the question of whether a reduction of the hourly cost might transform into holder license benefit and/or fare reduction

In terms of the recharging and swapping station, deepens in its costs in order to obtain a more accurate analysis.

This work has taken into account constant velocities, taxi supply and trip demand and leave beyond the scope of this research congestion scenarios with non-constant velocities or other variables. A future research should focus on how analyze the effects of heterogeneous areas.

This study has integrated different types of market, however, it would be interesting a modelling that joins in the same model different types of vehicles able in the same scenario by transitioning from standard vehicles with conventional fuels to electric and automated electric vehicles equations by following Carpenter (2013) who proposes a *return of investment for taxi companies transitioning to electric vehicles*.

The way to solve the optimum cost is obtaining the minimum value of a system of aggregated costs, however, some administrations solve this optimal not by the minimum but when the taxi cost is zero. It implies a small increase in the waiting and access times (Salanova, et al., 2015) and would be interesting to analyses this model for this solution.

References

Carpenter Tommy, Curtis Andrew R. and Keshav S. The Return On Investment for Taxi Companies Transitioning to Electric Vehicles [Journal] // University of Waterloo Technical Report CS-2011-20. - 2013.

Daganzo Carlos F. An Approximate Analytic Model of Many-to-Many Demand Responsive Transportation Systems [Journal] // Transportation Research. - 1978.

Daganzo Carlos F. Checkpoint Dial-a-Ride Systems [Journal] // Transportation Research, 18B. - Berkeley, California : Institute of Transportation Studies, University of California, 1984. - pp. 315-327.

Daganzo Carlos F. Public Transportation Systems: Basic Principles of System Design, Operations Planning and Real-Time Control [Book]. - Berkeley, California : Institu of Transportation Studies. University of Berkeley, 2010.

Fernández L. and De Cea J. F. Joaquin A diagrammatic analysis of the market for cruising taxis. Transportation Research Part E No. 42 pp. 498 – 526. [Journal]. - 2006.

Fernández L. J. E., de Cea and Malbran R. H. Demand Responsive Urban Public Transport System Design: Methodology and Application. Trasnportation Research Part A: 951 - 972 [Journal]. - 2008.

Geroliminis N. and Daganzo C.F. Existence of urban-scale macroscopic fundamental diagrams: Some experimental findings. Transportation Research Part B. 759-770 [Journal]. - 2008.

Greenberg H. An analysis of traffic flow. Operations Research Vol. 7. 78-85 [Journal]. - 1959.

Greenshield B.D. A study of traffic capacity. Highway Research Board Proceedings, 14 [Journal]. - 1935.

Kim H. Effect of taxi information system on efficiency and quality of taxi services. Transportation Research Record, 1903 [Journal]. - 2005.

Kittelson Transit Capacity and Quality of Service Manual. Transit cooperative research program report 100, 2nd edition [Report]. - 2003.

Lidicker Jeffrey, Timothy Lipman and Brett Williams Business Model for Subscription Service for Electric Vehicles Including Battery Swapping, for San Francisco Bay Area, California [Journal]. - San Francisco : Transportation Research, 2011.

Salanova J. [et al.] A review of the modeling of taxi services. Procedia Social and Behavioral Sciences 20, 150–161 [Journal]. - 2011.

Salanova Josep Maria and Estrada Miquel Modelling and Comparisson of Taxi Operational Modes: Case Study in Barcelona [Journal]. - 2015.

Sathaye N. and Kelley S. A review of the modelling of taxi services. Procedia and Behavioral Sciences 20, 150-161 [Journal]. - 2011.

Sathaye Nakul The Optimal Design and Cost Implications of Electric Vehicle Taxi [Journal] // Transportation Research Part B. - 2014. - pp. 264-283.

Schroeder Andreas and Traber Thure The Economics of Fast Charging Infrastructure for Electric Vehicles [Journal]. - Berlin, Germany : Energy Policy, 2012. - 43.

Schroeter J.R. A model of taxi service under fare structure and fleet size regulation. The Bell Journals of Economics, Vol. 14 [Journal]. - 1983.

Transport Centre d'Innovació del and Taxi Institut Metropolità del Obervatori del taxi. Document anual. - Barcelona : [s.n.], 2015.

UNFPA State of World Population 2007, Unleashing the Potential of Urban Growth [Book]. - 2007.
Yang H. and Yang T. Equilibrium properties of taxi markets with search frictions. Transportation Research Part B 45, 696-713 [Journal]. - 2011.
Yang H. [et al.] A macroscopic taxi model for passenger demand, taxi utilization and level of services. Transportation 27 [Journal]. - 2000.
Yang H. and Wong S.C. A network model of urban taxi services. Transportation Research Part B 32, 235-246 [Journal]. - 1998.

APPENDIX

1. SV TAXI SYSTEM FORMULATION

1.1. Introduction

This first part of the appendix will show the equations of the different costs: user, taxi, infrastructure and external one, as well as the distances and velocities required for them for the standard vehicles type.

1.2. Dispatching market

In this section the formulation of the conventional vehicles powered by fuel will be solved for the dispatching market.

1.2.1. Distances and velocities

The total distance ridden by a taxi in a trip or in a cycle will be, d,

$$d = d_D + d_Q + d_I \tag{A. 1}$$

In turn, the **dispatching distance**, d_D, what is the distance that takes a taxi to reach to the passenger who has dispatched is related with the uniformly distributed density all over the area of the empty taxis ready for a call. This distance takes into account that all the nearest taxi will be the one in the whole area that will be dispatched. For a circular region (Daganzo, 1978),

$$d_D = \frac{0.51r\sqrt{A}}{\sqrt{M_E}} \tag{A. 2}$$

And for a square area (Daganzo, 1978),

$$d_D = \frac{0.52r\sqrt{A}}{\sqrt{M_E}} \tag{A. 3}$$

This distance works when the fleet of a city is already known. In case the modelling of this city has to be done, the following approximation related to the in-vehicle distance can be considered (Salanova, et al., 2015),

$$d_D + d_I = 0.1d_Q \tag{A. 4}$$

Where,

$$d_D = d_I = 0.05 \cdot d_Q \tag{A. 5}$$

In order to obtain the average velocity, $\bar{v}$, and the average time, $\bar{t}$, per trip served,

$$P_Q = \frac{d_Q}{d} \tag{A. 6}$$

$$P_I = \frac{d_I}{d} \tag{A. 7}$$

$$P_D = \frac{d_D}{d} \tag{A. 8}$$

The average velocity servicing a trip, $\bar{v}$, is

$$\bar{v} = P_Q \cdot v_Q + P_I \cdot v_I + P_D \cdot v_D \tag{A. 9}$$

And the average time servicing a trip, $\bar{v}$, is

$$\bar{t} = \frac{d}{\bar{v}} \tag{A. 10}$$

1.2.2. Costs

In order to solve the one trip cost for one user in the dispatching market, the access time and the waiting time will be defined. Since users have to wait for the taxi dispatched without having to move by walking, the access time, T_A, will be zero,

$$T_A = 0 \tag{A. 11}$$

The waiting time, T_w, will take into account the time a user takes for making the dispatching order along with the distance that a dispatched taxi takes to reach the passenger will be (Salanova, et al., 2015),

$$T_w = \frac{0.4r}{\bar{v}\sqrt{\lambda_d - \lambda_u \cdot \bar{t}}} \tag{A. 12}$$

Therefore, the **one user cost for one trip is**,

$$Z_u^{trip} = \frac{0.4r}{\bar{v}\sqrt{\lambda_d - \lambda_u \cdot \bar{t}}} + T_Q + \frac{\bar{c}}{VoT} \tag{A. 13}$$

The **one user cost for all trip** is,

$$Z_u = \lambda_u \cdot A \cdot \left(\frac{0.4r}{\bar{v}\sqrt{\lambda_d - \lambda_u \cdot \bar{t}}} + T_Q + \frac{\bar{c}}{VoT} \right) \tag{A. 14}$$

The **one trip cost for one taxi**,

$$Z_t^{trip} = \frac{1}{\bar{n} \cdot VoT} \left[-\frac{\lambda_u}{\lambda_d} \cdot \bar{c} + \frac{\lambda_u}{\lambda_d} \cdot (1.1 \cdot d_Q) \cdot C_{km} + C_h \right] \tag{A. 15}$$

The **all trips cost for one taxi in one hour**,

$$Z_t^{all\ trips} = \frac{1}{VoT} \left[-\frac{\lambda_u}{\lambda_t} \cdot \bar{c} + \frac{\lambda_u}{\lambda_t} \cdot (1.1 \cdot d_Q) \cdot C_{km} + C_h \right] \tag{A. 16}$$

The **all trips cost for all taxis in one hour and in specific area**,

$$Z_t = \frac{\lambda_d \cdot A}{VoT} \left[-\frac{\lambda_u}{\lambda_d} \cdot \bar{c} + \frac{\lambda_u}{\lambda_d} \cdot (1.1 \cdot d_Q) \cdot C_{km} + C_h \right] \tag{A. 17}$$

The **external cost** will be zero as it is just considered for the hailing, dispatching-hailing and stand-hailing markets and the **infrastructure cost** will be as well null.

1.3. Hailing market

In this section the formulation of the conventional vehicles powered by fuel will be solved for the hailing market.

1.3.1. Distances and velocities

The total distance ridden by a taxi in a trip or in a cycle will be, d,

$$d = d_Q + d_I \tag{A. 18}$$

In this mode, taxis do not stop running and taking into consideration the framework of the modelling is 1 hour,

$$d = \bar{v} \cdot 1\ hour = \bar{v} \tag{A. 19}$$

The **idle time**, T_I,

$$T_I = \frac{(1 - \bar{n} \cdot T_Q)}{\bar{n}} \tag{A. 20}$$

The **idle distance**, d_i,

$$d_I = T_I \cdot v_I \tag{A. 21}$$

Consequently, the total distance, d,

$$d = d_Q + d_I \tag{A. 22}$$

$$P_Q = \frac{d_Q}{d} \tag{A. 23}$$

$$P_I = \frac{d_I}{d} \tag{A. 24}$$

The average velocity servicing a trip, $\bar{v}$, is

$$\bar{v} = P_Q \cdot v_Q + P_I \cdot v_I \tag{A. 25}$$

And the average time servicing a trip, $\bar{v}$, is

$$\bar{t} = \frac{d}{\bar{v}} \tag{A. 26}$$

1.3.2. Costs

In order to solve the one trip cost for one user in the hailing market, the access time and the waiting time will be defined. In this case, taxis will be hailed by the customers while taxis drive empty looking for a taxi, therefore, the access time, T_A,

$$T_A = 0 \tag{A. 27}$$

The waiting time (T_w) used in the hailing market will be (Fernández, et al., 2008),

$$T_W = \frac{K}{A \cdot \lambda_d - A \cdot \lambda_u \cdot \bar{t}} = \frac{1}{\bar{v} \cdot (\lambda_d - \lambda_u \cdot \bar{t})} \tag{A. 28}$$

Where (K) is the parameter that needs to be calibrated (Douglas 1972),

$$K = \frac{A}{\bar{v}} \tag{A. 29}$$

Therefore, the **one user cost for one trip** is,

$$Z_u^{trip} = \frac{1}{\bar{v} \cdot (\lambda_d - \lambda_u \cdot \bar{t})} + t_Q + \frac{\bar{c}}{VoT_u} \tag{A. 30}$$

The **all trips cost for all the user in a specific area**

$$Z_u = \lambda_u \cdot A \cdot \left[\frac{1}{\bar{v} \cdot (\lambda_d - \lambda_u \cdot \bar{t})} + t_Q + \frac{\bar{c}}{VoT_u}\right] \tag{A. 31}$$

The **one trip cost for one taxi**

$$Z_t^{trip} = \frac{1}{\bar{n} \cdot VoT}\left(-\frac{\lambda_u}{\lambda_d} \cdot \bar{c} + \frac{\lambda_u}{\lambda_d} \cdot \bar{v} \cdot C_{km} + C_h\right) \tag{A. 32}$$

The all trips cost for one taxi in one hour is,

$$Z_t^{all\ trips} = \frac{1}{VoT}\left(-\frac{\lambda_u}{\lambda_d} \cdot \bar{c} + \frac{\lambda_u}{\lambda_d} \cdot \bar{v} \cdot C_{km} + C_h\right) \tag{A. 33}$$

The all trips cost for all taxis in one hour and in specific area

$$Z_t = \frac{\lambda_d \cdot A}{VoT}\left(-\frac{\lambda_u}{\lambda_d} \cdot \bar{c} + \frac{\lambda_u}{\lambda_d} \cdot \bar{v} \cdot C_{km} + C_h\right) \tag{A. 34}$$

The **external cost** will be the same as explained at chapter 3 and the **infrastructure cost** will be 0.

1.4. Stand market

1.4.1. Distances and velocities

The total distance ridden by a taxi in a trip or in a cycle will be, d,

$$d = d_D + d_I \tag{A. 35}$$

The **stand distance**, d_S, (it can be also called idle distance, d_I)that takes an empty taxi to reach the stand is related with the uniformly distributed density all over the area of the stands. This distance takes into account that all the nearest stand. For a circular region (Daganzo, 1978),

$$d_S = d_I = \frac{0.51r\sqrt{A}}{\sqrt{s}} \tag{A. 36}$$

And for a square area (Daganzo, 1978),

$$d_S = d_I = \frac{0.52r\sqrt{A}}{\sqrt{s}} \tag{A. 37}$$

This distance works when the fleet of a city is already known. In case the modelling of this city has to be done, the following approximation related to the in-vehicle distance can be considered (Salanova, et al., 2015),

$$d_S = d_I = 0.05 d_Q \tag{A. 38}$$

In order to obtain the average velocity, $\bar{v}$, and the average time, $\bar{t}$, per trip served,

$$P_Q = \frac{d_Q}{d} \tag{A. 39}$$

$$P_s = \frac{d_S}{d} \tag{A. 40}$$

The average velocity servicing a trip, $\bar{v}$, is

$$\bar{v} = P_Q \cdot v_Q + P_S \cdot v_S \tag{A. 41}$$

And the average time servicing a trip, $\bar{v}$, is

$$\bar{t} = \frac{d}{\bar{v}} \tag{A. 42}$$

1.4.2. Costs

The access time, T_A,

$$s = \frac{A}{a^2} = A \cdot \left(\lambda_d - \lambda_u \cdot \frac{d}{v} \right) \tag{A. 43}$$

$$a = \frac{1}{\sqrt{\lambda_d - \lambda_u \cdot \bar{t}}} = \frac{1}{\sqrt{\lambda_d - \lambda_u \cdot \bar{t}}} \tag{A. 44}$$

$$T_A = \frac{1}{2 \cdot \overline{v_u} \sqrt{\lambda_d - \lambda_u \cdot \bar{t}}} \tag{A. 45}$$

On the other hand, the waiting time (T_w) will be,

$$T_w = 0 \tag{A. 46}$$

Therefore, the **one user cost for one trip is**,

$$Z_u^{trip} = \frac{1}{2 \cdot \overline{v_u} \sqrt{\lambda_d - \lambda_u \cdot \bar{t}}} + T_Q + \frac{\bar{c}}{VoT} \tag{A. 47}$$

The **all trips cost for all the user in a specific area**,

$$Z_u = \lambda_u \cdot A \cdot \left(\frac{1}{2 \cdot \overline{v_u}\sqrt{\lambda_d - \lambda_u \cdot \bar{t}}} + T_Q + \frac{\bar{c}}{VoT} \right) \tag{A. 48}$$

The **one trip cost for one taxi**,

$$Z_t^{trip} = \frac{1}{\bar{n} \cdot VoT} \left[-\frac{\lambda_u}{\lambda_d} \cdot \bar{c} + \frac{\lambda_u}{\lambda_d} \cdot (1.05 \cdot d_Q) \cdot C_{km} + C_h \right] \tag{A. 49}$$

The **all trips cost for one taxi in one hour**,

$$Z_t^{all\ trips} = \frac{1}{VoT} \left[-\frac{\lambda_u}{\lambda_d} \cdot \bar{c} + \frac{\lambda_u}{\lambda_d} \cdot (1.05 \cdot d_Q) \cdot C_{km} + C_h \right] \tag{A. 50}$$

The **all trips cost for all taxis in one hour and in specific area,**

$$Z_t = \frac{\lambda_d \cdot A}{VoT} \left[-\frac{\lambda_u}{\lambda_d} \cdot \bar{c} + \frac{\lambda_u}{\lambda_d} \cdot (1.05 \cdot d_Q) \cdot C_{km} + C_h \right] \tag{A. 51}$$

The **external cost** will be zero as it is considered for the hailing, dispatching-hailing and stand-hailing markets and the **infrastructure cost** will be the stand one.

1.5. Dispatching-hailing market

1.5.1. Distances and velocities

The dispatching-hailing total distance will be solved by combination of the dispatching and hailing by different parts and afterwards combined by the parameter, γ.

Dispatching part

The total distance ridden by a taxi in a trip or in a cycle will be, d,

$$d_{DISP} = d_D + d_Q + d_I \tag{A. 52}$$

Where idle and dispatching distances has been explained in prior sections.

In order to obtain the average velocity, $\bar{v}$, and the average time, $\bar{t}$, per trip served,

$$P_{DISP,Q} = \frac{d_Q}{d_{DISP}} \tag{A. 53}$$

$$P_{DISP,I} = \frac{d_I}{d_{DISP}} \tag{A. 54}$$

$$P_{DISP,D} = \frac{d_D}{d_{DISP}} \tag{A. 55}$$

The average velocity servicing a trip, $\bar{v}$, is

$$\bar{v}_{DISP} = P_{DISP,Q} \cdot v_Q + P_{DISP,I} \cdot v_I + P_{DISP,D} \cdot v_D \tag{A. 56}$$

Hailing part

The total distance ridden by a taxi in a trip or in a cycle will be, d,

$$d_{HAIL} = d_Q + d_{HAIL,I} \tag{A. 57}$$

Where idle and dispatching distances has been explained in prior sections.

The **idle time** in the hailing part, $T_{HAIL,I}$, (

$$T_{HAIL,I} = \frac{(1 - \bar{n} \cdot T_Q)}{\bar{n}} \tag{A. 58}$$

The **idle distance** in the hailing part, $d_{HAIL,I}$,

$$d_{HAIL,I} = T_{HAIL,I} \cdot v_I \tag{A. 59}$$

$$P_{HAIL,Q} = \frac{d_Q}{d_{HAIL}} \tag{A. 60}$$

$$P_{HAIL,I} = \frac{d_{HAIL,I}}{d_{HAIL}} \tag{A. 61}$$

The average velocity servicing a trip, $\bar{v}$, is

$$\bar{v}_{HAIL} = P_{HAIL,Q} \cdot v_Q + P_{HAIL,I} \cdot v_I \tag{A. 62}$$

Dispatching-hailing combination

So,

$$\bar{v} = \bar{v}_{DISP} \cdot (1 - \gamma) + \bar{v}_{HAIL} \cdot \gamma \tag{A. 63}$$

The total distance ridden by a taxi in a trip or in a cycle will be, d,

$$d = (1 - \gamma) \cdot d_{DISP} + \gamma \cdot d_{HAIL} \tag{A. 64}$$

And the total idle distance,

$$d_I = (1 - \gamma) \cdot 0.1 \cdot d_Q + \gamma \cdot d_{HAIL,I} \tag{A. 65}$$

And the average time servicing a trip, $\bar{v}$, is

$$\bar{t} = \frac{d}{\bar{v}} \tag{A. 66}$$

1.5.2. Costs

The **one trip cost for one user**,

$$Z_u^{trip} = (1-\gamma)\left(\frac{0.4r}{\bar{v}\sqrt{\lambda_d - \lambda_d \cdot \bar{t}}}\right) + \gamma\left(\frac{1}{\bar{v}(\lambda_d - \lambda_u \cdot \bar{t})}\right) + d_Q + \frac{\bar{c}}{VoT} \tag{A. 67}$$

The **all trips cost for all the user in a specific area**,

$$Z_u = \lambda_u \cdot A \cdot \left[(1-\gamma)\left(\frac{0.4r}{\bar{v}\sqrt{\lambda_d - \lambda_d \cdot \bar{t}}}\right) + \gamma\left[\frac{1}{\bar{v}(\lambda_d - \lambda_u \cdot \bar{t})}\right] + d_Q + \frac{\bar{c}}{VoT}\right] \tag{A. 68}$$

The **one trip cost for one taxi**,

$$Z_t^{trip} = \frac{1}{\bar{n} \cdot VoT}\left[-\frac{\lambda_u}{\lambda_d} \cdot \bar{c} + \frac{\lambda_u}{\lambda_d} \cdot [(1-\gamma) \cdot d_{DISP} + \gamma \cdot d_{HAIL}] \cdot C_{km} + C_h\right] \tag{A. 69}$$

The **all trips cost for one taxi in one hour**,

$$Z_t^{all\ trips} = \frac{1}{VoT}\left[-\frac{\lambda_u}{\lambda_d} \cdot \bar{c} + \frac{\lambda_u}{\lambda_d} \cdot [(1-\gamma) \cdot d_{DISP} + \gamma \cdot d_{HAIL}] \cdot C_{km} + C_h\right] \tag{A. 70}$$

The **all trips cost for all taxis in one hour and in specific area**,

$$Z_t = \frac{\lambda_d \cdot A}{VoT}\left[-\frac{\lambda_u}{\lambda_d} \cdot \bar{c} + \frac{\lambda_u}{\lambda_d} \cdot [(1-\gamma) \cdot d_{DISP} + \gamma \cdot d_{HAIL}] \cdot C_{km} + C_h\right] \tag{A. 71}$$

The **external cost** will be the same as explained at chapter 3 and the **infrastructure cost** will be 0.

1.6. Stand-hailing market

1.6.1. Distances and velocities

The stand-hailing total distance will be solved by combination of the dispatching and hailing by different parts and afterwards combined by the parameter, γ.

Stand part

The total distance ridden by a taxi in a trip or in a cycle will be, d,

$$d_{STAND} = d_Q + d_I \tag{A. 72}$$

Where idle distance has been explained in prior sections.

In order to obtain the average velocity, $\bar{v}$, and the average time, $\bar{t}$, per trip served,

$$P_{STAND,Q} = \frac{d_Q}{v_Q} \tag{A. 73}$$

$$P_{STAND,S} = \frac{d_I}{v_S} \tag{A. 74}$$

The average velocity servicing a trip, $\bar{v}$, is

$$\bar{v}_{STAND} = P_{STAND,Q} \cdot v_Q + P_{STAND,S} \cdot v_S \tag{A. 75}$$

Hailing part

The total distance ridden by a taxi in a trip or in a cycle will be, d,

$$d_{HAIL} = d_Q + d_{HAIL,I} \tag{A. 76}$$

Where idle and dispatching distances has been explained in prior sections.

The **idle time** in the hailing part, $T_{HAIL,I}$,

$$T_{HAIL,I} = \frac{(1 - \bar{n} \cdot T_Q)}{\bar{n}} \tag{A. 77}$$

The **idle distance** in the hailing part, $d_{HAIL,I}$,

$$d_{HAIL,I} = T_{HAIL,I} \cdot v_I \tag{A. 78}$$

$$P_{HAIL,Q} = \frac{d_Q}{d_{HAIL}} \tag{A. 79}$$

$$P_{HAIL,I} = \frac{d_{HAIL,I}}{d_{HAIL}} \tag{A. 80}$$

The average velocity servicing a trip, $\bar{v}$, is

$$\bar{v}_{HAIL} = P_{HAIL,Q} \cdot v_Q + P_{HAIL,I} \cdot v_I \tag{A. 81}$$

So,

$$\bar{v} = \bar{v}_{STAND} \cdot (1 - \gamma) + \bar{v}_{HAIL} \cdot \gamma \tag{A. 82}$$

The total distance ridden by a taxi in a trip or in a cycle will be, d,

$$d = (1-\gamma)\cdot d_{STAND} + \gamma \cdot d_{HAIL} \tag{A. 83}$$

And the total idle distance,

$$d_I = (1-\gamma)\cdot 0.05 \cdot d_Q + \gamma \cdot d_{HAIL,I} \tag{A. 84}$$

And the average time servicing a trip, $\bar{v}$, is

$$\bar{t} = \frac{d}{\bar{v}} \tag{A. 85}$$

1.6.2. Costs

The **one trip for one user**

$$Z_u^{trip} = (1-\gamma)\frac{1}{2\cdot \overline{v_u}\sqrt{\lambda_d - \lambda_u \cdot \bar{t}}} + \gamma\left[\frac{1}{\bar{v}(\lambda_d - \lambda_u \cdot \bar{t})}\right] + T_Q + \frac{\bar{c}}{VoT} \tag{A. 86}$$

The **all trips cost for all the user in a specific area**

$$Z_u = \lambda_u \cdot A \cdot \left((1-\gamma)\frac{1}{2\cdot \overline{v_u}\sqrt{\lambda_d - \lambda_u \cdot \bar{t}}} + \gamma\left[\frac{1}{\bar{v}(\lambda_d - \lambda_u \cdot \bar{t})}\right] + T_Q + \frac{\bar{c}}{VoT}\right) \tag{A. 87}$$

The **one trip cost for one taxi**

$$Z_t^{trip} = \frac{1}{\bar{n}\cdot VoT}\left[-\frac{\lambda_u}{\lambda_d}\cdot \bar{c} + \frac{\lambda_u}{\lambda_d}\cdot [(1-\gamma)\cdot d_{STAND} + \gamma \cdot d_{HAIL}]\cdot C_{km} + C_h\right] \tag{A. 88}$$

The **all trips cost for one taxi in one hour**

$$Z_t^{all\ trips} = \frac{1}{VoT}\left[-\frac{\lambda_u}{\lambda_d}\cdot \bar{c} + \frac{\lambda_u}{\lambda_d}\cdot [(1-\gamma)\cdot d_{STAND} + \gamma \cdot d_{HAIL}]\cdot C_{km} + C_h\right] \tag{A. 89}$$

The **all trips cost for all taxis in one hour and in specific area**

$$Z_t = \frac{\lambda_d \cdot A}{VoT}\left[-\frac{\lambda_u}{\lambda_d}\cdot \bar{c} + \frac{\lambda_u}{\lambda_d}\cdot [(1-\gamma)\cdot d_{STAND} + \gamma \cdot d_{HAIL}]\cdot C_{km} + C_h\right] \tag{A. 90}$$

The **external cost** will be the same as explained at chapter 3 and the **infrastructure cost** will be 0.

2. BEV TAXI SYSTEM FORMULATION

2.1. Introduction

This second part of the appendix will show the equations of the different costs: user, taxi, infrastructure and external one, as well as the distances and velocities required for them for the battery electric vehicles and driverless battery electric vehicles types.

2.2. Dispatching market

2.2.1. Distances, velocities and charging time

The total distance ridden by a taxi in a trip or in a cycle will be, d,

$$d = d_D + d_Q + d_I + \frac{2d_C}{N} \quad \text{(A. 91)}$$

The equation above will represent the dispatching distance, d_D, the in-vehicle travel distance, d_Q, the idle distance, d_I, the distance to the charging station divided by the range, d_C/N and the distance ridden from the charging station driving outwards to lower density areas, d_C/N.

In order to obtain the average velocity, $\bar{v}$, and the average time, $\bar{t}$, per trip served,

$$P_Q = \frac{d_Q}{d} \quad \text{(A. 92)}$$

$$P_I = \frac{d_I}{d} \quad \text{(A. 93)}$$

$$P_D = \frac{d_D}{d} \quad \text{(A. 94)}$$

$$P_C = \frac{2 \cdot d_C}{d \cdot N} \quad \text{(A. 95)}$$

The average velocity servicing a trip, $\bar{v}$, is

$$\bar{v} = P_Q \cdot v_Q + P_I \cdot v_I + P_D \cdot v_D + P_C \cdot v_C \quad \text{(A. 96)}$$

And the average time servicing a trip, $\bar{v}$, is

$$\bar{t} = \frac{d}{\bar{v}} \quad \text{(A. 97)}$$

2.2.2. Costs

The **one trip for one user** and the **all trips cost for all the user in a specific area** will be the same as for SV

The **one trip cost for one taxi**,

$$Z_t^{trip} = \frac{1}{\bar{n} \cdot VoT}\left[-\frac{\lambda_u}{\lambda_d} \cdot \bar{c} + \frac{\lambda_u}{\lambda_d} \cdot (1.1 \cdot d_Q + \frac{2d_C}{N}) \cdot C_{km} + C_h\right] \tag{A. 98}$$

The **all trips cost for one taxi in one hour**,

$$Z_t^{all\ trips} = \frac{1}{VoT}\left[-\frac{\lambda_u}{\lambda_d} \cdot \bar{c} + \frac{\lambda_u}{\lambda_d} \cdot (1.1 \cdot d_Q + \frac{2d_C}{N}) \cdot C_{km} + C_h\right] \tag{A. 99}$$

The **all trips cost for all taxis in one hour and in specific area**,

$$Z_t = \frac{\lambda_d \cdot A}{VoT}\left[-\frac{\lambda_u}{\lambda_d} \cdot \bar{c} + \frac{\lambda_u}{\lambda_d} \cdot (1.1 \cdot d_Q + \frac{2d_C}{N}) \cdot C_{km} + C_h\right] \tag{A. 100}$$

The **external cost** will be zero as it is just considered for the hailing, dispatching-hailing and stand-hailing markets and the **infrastructure cost** will be the swapping or the recharging by plug-in station.

2.3. Hailing market

2.3.1. Distances, velocities and charging time

The total distance ridden by a taxi in a trip or in a cycle will be, d,

$$d = d_Q + d_I + \frac{2d_C}{N} \tag{A. 101}$$

In this mode, taxis do not stop running and taking into consideration the framework of the modelling is 1 hour,

$$d = \bar{v} \cdot 1\ hour = \bar{v} \tag{A. 102}$$

The **idle time**, T_I,

$$T_I = \frac{(1 - \bar{n} \cdot T_Q - \bar{n} \cdot \frac{2d_C}{N})}{\bar{n}} \tag{A. 103}$$

The **idle distance**, d_i,

$$d_I = T_I \cdot v_I \tag{A. 104}$$

Consequently, once known the total distance, d,

$$P_Q = \frac{d_Q}{d} \tag{A. 105}$$

$$P_I = \frac{d_I}{d} \tag{A. 106}$$

$$P_C = \frac{2 \cdot d_C}{d \cdot N} \tag{A. 107}$$

The average velocity servicing a trip, $\bar{v}$, is

$$\bar{v} = P_Q \cdot v_Q + P_I \cdot v_I + P_C \cdot v_C \tag{A. 108}$$

And the average time servicing a trip, $\bar{v}$, is

$$\bar{t} = \frac{d}{\bar{v}} \tag{A. 109}$$

2.3.2. Costs

The **one trip for one user cost** and the **all trips cost for all the user in a specific area** will be the same as for SV.

The one trip cost for one taxi,

$$Z_t^{trip} = \frac{1}{\bar{n} \cdot VoT}\left(-\frac{\lambda_u}{\lambda_d} \cdot \bar{c} + \frac{\lambda_u}{\lambda_d} \cdot \bar{v} \cdot C_{km} + C_{\hbar}\right) \tag{A. 110}$$

The **all trips cost for one taxi in one hour**,

$$Z_t^{all\ trips} = \frac{1}{VoT}\left(-\frac{\lambda_u}{\lambda_d} \cdot \bar{c} + \frac{\lambda_u}{\lambda_d} \cdot \bar{v} \cdot C_{km} + C_h\right) \tag{A. 111}$$

The **all trips cost for all taxis in one hour and in specific area**,

$$Z_t = \frac{\lambda_d \cdot A}{VoT}\left(-\frac{\lambda_u}{\lambda_d} \cdot \bar{c} + \frac{\lambda_u}{\lambda_d} \cdot \bar{v} \cdot C_{km} + C_h\right) \tag{A. 112}$$

The **external cost** will be considered as explained at chapter 3 and the **infrastructure cost** will be the swapping or the recharging by plug-in station.

2.4. Stand market

2.4.1. Distances, velocities and charging time

The total distance ridden by a taxi in a trip or in a cycle for the stand market with ICEV and HEV will be, d,

$$d = d_Q + d_S + \frac{2d_C}{N} \tag{A. 113}$$

In order to obtain the average velocity, $\bar{v}$, and the average time, $\bar{t}$, per trip served,

$$P_Q = \frac{d_Q}{d} \tag{A. 114}$$

$$P_s = \frac{d_S}{d} \tag{A. 115}$$

$$P_C = \frac{2 \cdot d_C}{d \cdot N} \tag{A. 116}$$

The average velocity servicing a trip, $\bar{v}$, is

$$\bar{v} = P_Q \cdot v_Q + P_S \cdot v_s + P_C \cdot v_C \tag{A. 117}$$

And the average time servicing a trip, $\bar{v}$, is

$$\bar{t} = \frac{d}{\bar{v}} \tag{A. 118}$$

2.4.2. Costs

The **one trip for one user** and the **all trips cost for all the user in a specific area** will be the same as for SV

The **one trip cost for one taxi**,

$$Z_t^{trip} = \frac{1}{\bar{n} \cdot VoT}\left[-\frac{\lambda_u}{\lambda_d} \cdot \bar{c} + \frac{\lambda_u}{\lambda_d} \cdot \left(1.05 \cdot d_Q + \frac{2d_C}{N}\right) \cdot C_{km} + C_h\right] \tag{A. 119}$$

The **all trips cost for one taxi in one hour**,

$$Z_t^{all\ trips} = \frac{1}{VoT}\left[-\frac{\lambda_u}{\lambda_d} \cdot \bar{c} + \frac{\lambda_u}{\lambda_d} \cdot \left(1.05 \cdot d_Q + \frac{2d_C}{N}\right) \cdot C_{km} + C_h\right] \tag{A. 120}$$

The **all trips cost for all taxis in one hour and in specific area**,

$$Z_t = \frac{\lambda_d \cdot A}{VoT}\left[-\frac{\lambda_u}{\lambda_d} \cdot \bar{c} + \frac{\lambda_u}{\lambda_d} \cdot \left(1.05 \cdot d_Q + \frac{2d_C}{N}\right) \cdot C_{km} + C_h\right] \tag{A. 121}$$

The **external cost** will be zero as it is just considered for the hailing, dispatching-hailing and stand-hailing markets and the **infrastructure cost** will be the swapping or the recharging by plug-in station.

2.5. Dispatching-hailing market

2.5.1. Distances, velocities and charging time

The dispatching-hailing total distance will be solved by combination of the dispatching and hailing by different parts and afterwards combined by the parameter, γ.

Dispatching part

The total distance ridden by a taxi in a trip or in a cycle will be, d,

$$d_{DISP} = d_D + d_Q + d_I + \frac{2d_C}{N} \tag{A. 122}$$

In order to obtain the average velocity, $\bar{v}$, and the average time, $\bar{t}$, per trip served,

$$P_{DISP,Q} = \frac{d_Q}{d_{DISP}} \tag{A. 123}$$

$$P_{DISP,I} = \frac{d_{DISP,I}}{d_{DISP}} \tag{A. 124}$$

$$P_{DISP,D} = \frac{d_{DISP,D}}{d_{DISP}} \tag{A. 125}$$

$$P_{DISP,C} = \frac{2 \cdot d_C}{N \cdot d_{DISP}} \tag{A. 126}$$

The average velocity servicing a trip, $\bar{v}$, is

$$\bar{v}_{DISP} = P_{DISP,Q} \cdot v_Q + P_{DISP,I} \cdot v_I + P_{DISP,D} \cdot v_D + P_{DISP,C} \cdot v_C \tag{A. 127}$$

Hailing part

The total distance ridden by a taxi in a trip or in a cycle will be, d,

$$d_{HAIL} = d_Q + d_{HAIL,I} \tag{A. 128}$$

Where idle and dispatching distances has been explained in prior sections.

The **idle time** in the hailing part, $T_{HAIL,I}$,

$$T_{HAIL,I} = \frac{(1 - \bar{n} \cdot T_Q - \bar{n} \cdot \frac{2d_C}{N})}{\bar{n}} \tag{A. 129}$$

The **idle distance** in the hailing part, $d_{HAIL,I}$,

$$d_{HAIL,I} = T_{HAIL,I} \cdot v_I \tag{A. 130}$$

$$P_{HAIL,Q} = \frac{d_Q}{d_{HAIL}} \tag{A. 131}$$

$$P_{HAIL,I} = \frac{d_{HAIL,I}}{d_{HAIL}} \tag{A. 132}$$

$$P_{HAIL,C} = \frac{d_{HAIL,C}}{d_{HAIL}} \tag{A. 133}$$

The average velocity servicing a trip, $\bar{v}$, is

$$\bar{v}_{HAIL} = P_{HAIL,Q} \cdot v_Q + P_{HAIL,I} \cdot v_I + P_{HAIL,C} \cdot v_C \tag{A. 134}$$

Dispatching-hailing combination

So,

$$\bar{v} = \bar{v}_{DISP} \cdot (1-\gamma) + \bar{v}_{HAIL} \cdot \gamma \tag{A. 135}$$

The total distance ridden by a taxi in a trip or in a cycle will be, d,

$$d = (1-\gamma) \cdot d_{DISP} + \gamma \cdot d_{HAIL} \tag{A. 136}$$

And the total idle distance,

$$d_I = (1-\gamma) \cdot 0.05 \cdot d_Q + \gamma \cdot d_{HAIL,I} \tag{A. 137}$$

And the average time servicing a trip, $\bar{v}$, is

$$\bar{t} = \frac{d}{\bar{v}} \tag{A. 138}$$

2.5.2. Costs

The **one trip for one user** and the **all trips cost for all the user in a specific area** will be the same as for SV

The **one trip cost for one taxi**,

$$Z_t^{trip} = \frac{1}{\bar{n} \cdot VoT}\left[-\frac{\lambda_u}{\lambda_d} \cdot \bar{c} + \frac{\lambda_u}{\lambda_d} \cdot [(1-\gamma) \cdot d_{DISP} + \gamma \cdot d_{HAIL}] \cdot C_{km} + C_h\right] \tag{A. 139}$$

The **all trips cost for one taxi in one hour**,

$$Z_t^{all\ trips} = \frac{1}{VoT}\left[-\frac{\lambda_u}{\lambda_d} \cdot \bar{c} + \frac{\lambda_u}{\lambda_d} \cdot [(1-\gamma) \cdot d_{DISP} + \gamma \cdot d_{HAIL}] \cdot C_{km} + C_h\right] \tag{A. 140}$$

All trips cost for all taxis in one hour and in specific area

$$Z_t = \frac{\lambda_d \cdot A}{VoT}\left[-\frac{\lambda_u}{\lambda_d} \cdot \bar{c} + \frac{\lambda_u}{\lambda_d} \cdot [(1-\gamma) \cdot d_{DISP} + \gamma \cdot d_{HAIL}] \cdot C_{km} + C_h\right] \tag{A. 141}$$

The **external cost** will be considered as explained at chapter 3 and the **infrastructure cost** will be the swapping or the recharging by plug-in station.

2.6. Stand-hailing market

2.6.1. Distances, velocities and charging time

The stand-hailing total distance will be solved by combination of the dispatching and hailing by different parts and afterwards combined by the parameter, γ.

Stand part

The total distance ridden by a taxi in a trip or in a cycle will be, d,

$$d_{STAND} = d_Q + d_I + \frac{2d_C}{N} \tag{A. 142}$$

Where idle distance has been explained in prior sections.

In order to obtain the average velocity, $\bar{v}$, and the average time, $\bar{t}$, per trip served,

$$P_{STAND,Q} = \frac{d_{STAND,Q}}{d_{STAND}} \tag{A. 143}$$

$$P_{STAND,I} = \frac{d_{STAND,I}}{d_{STAND}} \tag{A. 144}$$

$$P_{STAND,C} = \frac{2 \cdot d_C}{N \cdot d_{STAND}} \tag{A. 145}$$

The average velocity servicing a trip, $\bar{v}$, is

$$\bar{v}_{STAND} = P_{STAND,Q} \cdot v_Q + P_{STAND,I} \cdot v_I + P_{STAND,C} \cdot v_C \tag{A. 146}$$

Hailing part

The total distance ridden by a taxi in a trip or in a cycle will be, d,

$$d_{HAIL} = d_Q + d_{HAIL,I} \tag{A. 147}$$

Where idle and dispatching distances has been explained in prior sections.

The **idle time** in the hailing part, $T_{HAIL,I}$,

$$T_{HAIL,I} = \frac{(1 - \bar{n} \cdot T_Q - \bar{n} \cdot \frac{2d_C}{N})}{\bar{n}} \tag{A. 148}$$

The **idle distance** in the hailing part, $d_{HAIL,I}$,

$$d_{HAIL,I} = T_{HAIL,I} \cdot v_I \tag{A. 149}$$

$$P_{HAIL,Q} = \frac{d_Q}{d_{HAIL}} \tag{A. 150}$$

$$P_{HAIL,C} = \frac{2 \cdot d_C}{N \cdot d_{HAIL}} \tag{A. 151}$$

The average velocity servicing a trip, $\bar{v}$, is

$$\bar{v}_{HAIL} = P_{HAIL,Q} \cdot v_Q + P_{HAIL,I} \cdot v_I + +P_{HAIL,C} \cdot v_C \tag{A. 152}$$

So,

$$\bar{v} = \bar{v}_{STAND} \cdot (1 - \gamma) + \bar{v}_{HAIL} \cdot \gamma \tag{A. 153}$$

The total distance ridden by a taxi in a trip or in a cycle will be, d,

$$d = (1 - \gamma) \cdot d_{STAND} + \gamma \cdot d_{HAIL} \tag{A. 154}$$

And the total idle distance,

$$d_I = (1 - \gamma) \cdot 0.05 \cdot d_Q + \gamma \cdot d_{HAIL,I} \tag{A. 155}$$

And the average time servicing a trip, $\bar{v}$, is

$$\bar{t} = \frac{d}{\bar{v}} \tag{A. 156}$$

2.6.2. Costs

The **one trip for one user** and the **all trips cost for all the user in a specific area** will be the same as for SV

The **one trip cost for one taxi**,

$$Z_t^{trip} = \frac{1}{\bar{n} \cdot VoT}\left[-\frac{\lambda_u}{\lambda_d} \cdot c + \frac{\lambda_u}{\lambda_d} \cdot d \cdot C_{km} + C_h\right] \tag{A. 157}$$

The **all trips cost for one taxi in one hour**,

$$Z_t^{all\ trips} = Z_d^{trip} \cdot \bar{n} \tag{A. 158}$$

The **all trips cost for all taxis in one hour and in specific area**,

$$Z_t = \lambda_d \cdot A \cdot Z_d^{all\ trips} \tag{A. 159}$$

The **external cost** will be considered as explained at chapter 3 and the **infrastructure cost** will be the swapping or the recharging by plug-in station.